The basics of graphene and its Raman spectroscopy

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Outline:

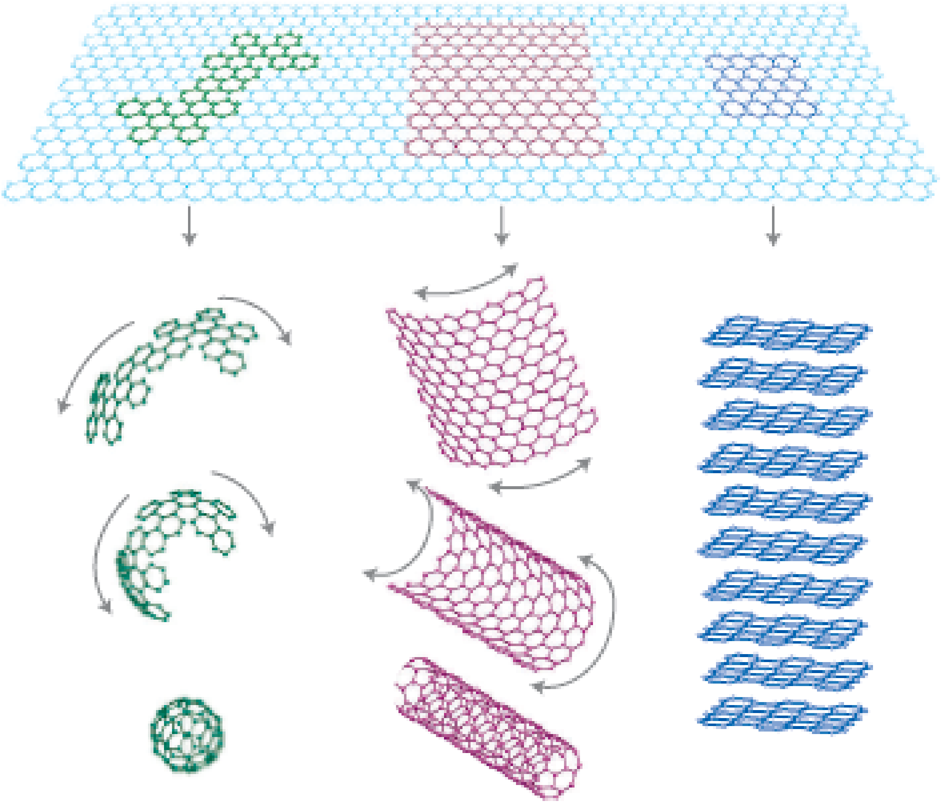
1. Structure and electronic properties
2. Optical properties: infrared absorption
3. Optical properties: Raman scattering

“Mother of all graphitic forms”

strong sp^2 bonds

↓

very few defects



2D graphene

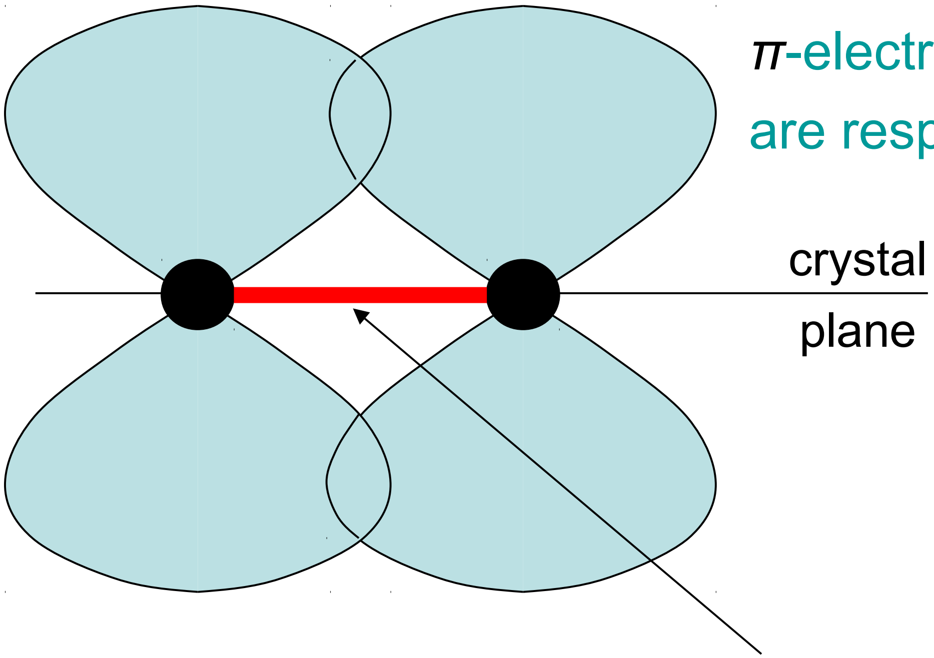
0D fullerenes

1D nanotubes

quasi-2D graphite

A. K. Geim and K. S. Novoselov, Nature Materials **6**, 183 (2007)

σ- and π-bonds

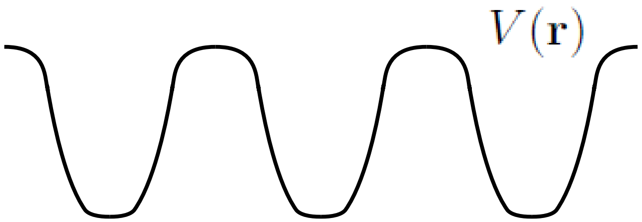


π-electrons (1 per atom)
are responsible for the conduction

sp^2-hybridized electrons
form tight σ-bonds
connecting each carbon atom
to its three neighbors

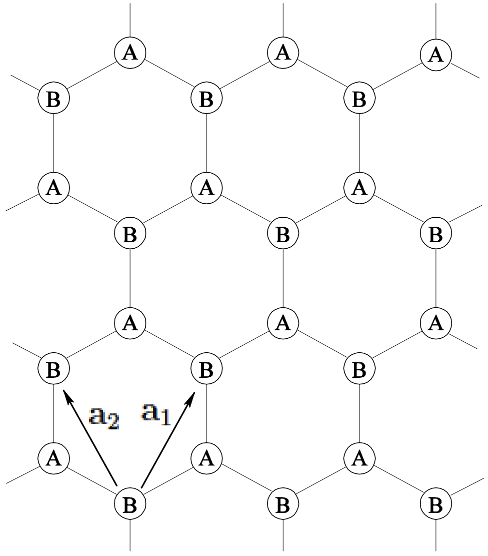
Bloch states in a periodic potential

$$\left[-\frac{\nabla^2}{2m}+V(\mathbf{r})\right]\psi(\mathbf{r})=\epsilon\,\psi(\mathbf{r})$$



$$V(\mathbf{r}+\mathbf{a}_i)=V(\mathbf{r})$$

$\mathbf{a}_1, \mathbf{a}_2$ - lattice vectors

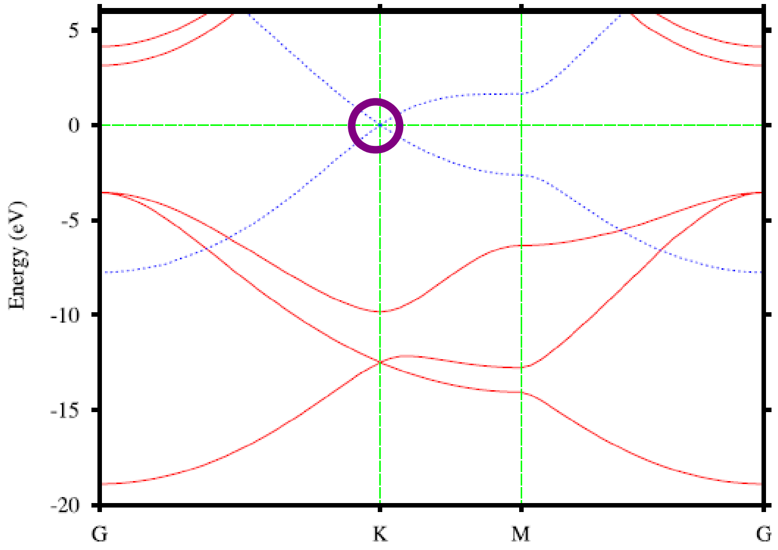


Bloch theorem: $\psi(\mathbf{r})=e^{i\mathbf{k}\mathbf{r}}\,u_{b\mathbf{k}}(\mathbf{r})$

quantum numbers $\left\{\begin{array}{l}\mathbf{k}\ \text{in the first Brillouin zone}\\ b\ \ \text{band index}\end{array}\right.$

$u_{b\mathbf{k}}(\mathbf{r}+\mathbf{a}_i)=u_{b\mathbf{k}}(\mathbf{r})$ periodic Bloch function, found from the Schrödinger equation in a single unit cell

$\epsilon=\epsilon_{b\mathbf{k}}$ band structure



from
Boukhvalov,
Katsnelson,
Lichtenstein
PRB 77, 035427 (2008)

Crystal symmetry

$$\left[-\frac{\nabla^2}{2m} - \frac{i\mathbf{k}}{m}\cdot\nabla + V(\mathbf{r})\right]u(\mathbf{r}) = \left(\epsilon - \frac{k^2}{2m}\right)u(\mathbf{r})$$

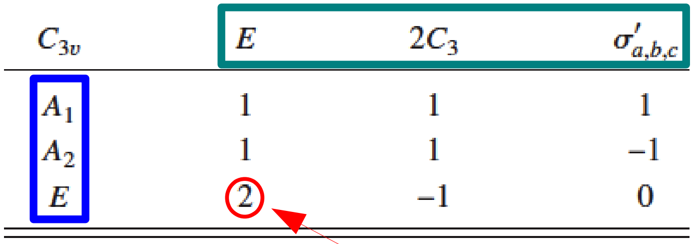
at K point: ***C_{3v}***

- rotation by $2\pi/3$
- mirror reflection

C_{6v}

- rotation by $\pi/3$
- mirror reflection

C_{3v} character table:



C_{3v}	E	$2C_3$	$\sigma'_{a,b,c}$
A_1	1	1	1
A_2	1	1	−1
E	2	−1	0

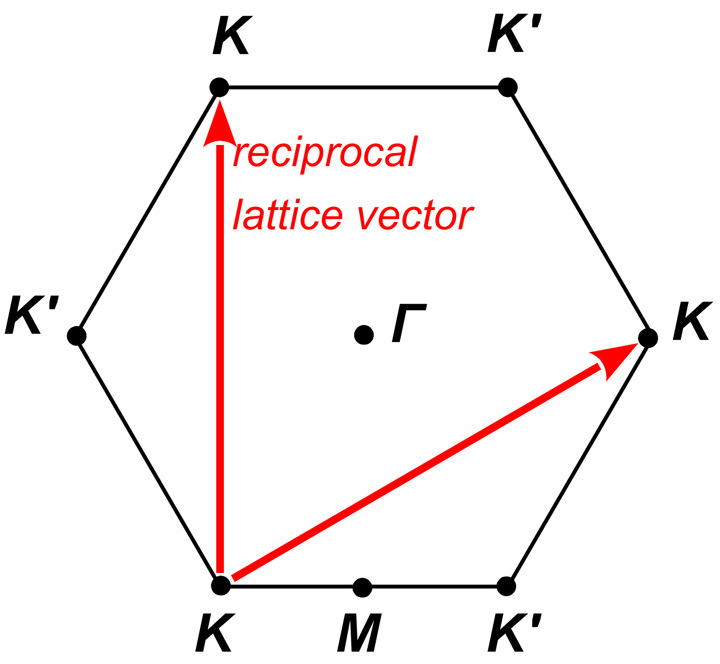
symmetry operations

irreducible representations

2-dimensional representation

Doubly degenerate states at the ***K*** point are allowed by the symmetry

first Brillouin zone:

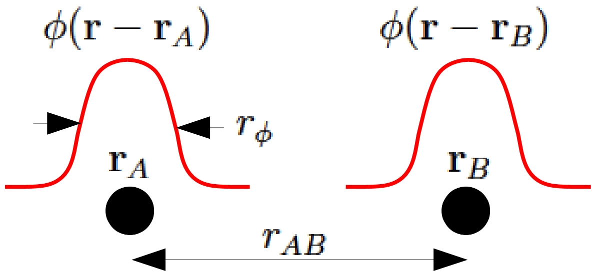


Time reversal symmetry

Additional degeneracy ***K*** ↔ ***K'***

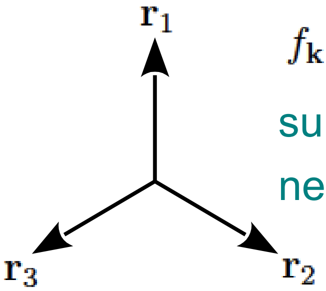
Tight-binding model

atomic π orbitals



Schrödinger equation:

$$\begin{bmatrix} \epsilon & \gamma_0 f_{\mathbf{k}} \\ \gamma_0 f_{\mathbf{k}}^* & \epsilon \end{bmatrix} \begin{bmatrix} C_{A\mathbf{k}} \\ C_{B\mathbf{k}} \end{bmatrix} = 0$$



$$f_{\mathbf{k}} \equiv \sum_{\alpha=1,2,3} e^{i\mathbf{k}\mathbf{r}_\alpha}$$

sum over nearest neighbors

$$\boxed{\epsilon_{\mathbf{k}} = \pm\gamma_0 |f_{\mathbf{k}}|}$$

P. R. Wallace, *Phys. Rev.* **71**, 622 (1947)

$$\psi(\mathbf{r}) = \sum_j \left[C_{A\mathbf{k}}\,\phi(\mathbf{r}-\mathbf{r}_{Aj})\,e^{i\mathbf{k}\mathbf{r}_{Aj}} + C_{B\mathbf{k}}\,\phi(\mathbf{r}-\mathbf{r}_{Bj})\,e^{i\mathbf{k}\mathbf{r}_{Bj}}\right]$$

sum over unit cells

diagonal matrix element:

$$\int \phi(\mathbf{r}-\mathbf{r}_A)\left(-\frac{\nabla^2}{2m}+V\right)\phi(\mathbf{r}-\mathbf{r}_A)\,d^3\mathbf{r} = \epsilon_\pi \to 0$$

atomic level: choose as the origin

nearest-neighbor matrix element:

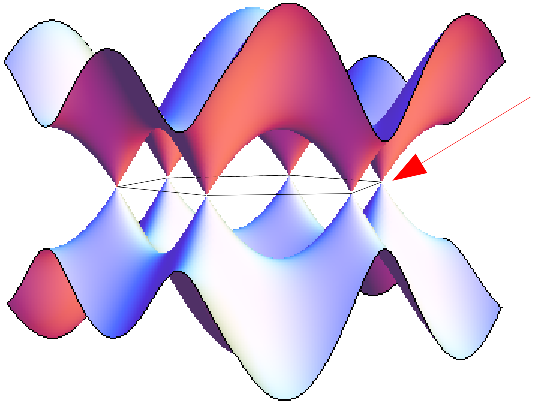
$$\int \phi(\mathbf{r}-\mathbf{r}_B)\left(-\frac{\nabla^2}{2m}+V\right)\phi(\mathbf{r}-\mathbf{r}_A)\,d^3\mathbf{r} = -\gamma_0$$

$$\boxed{\gamma_0 \approx 3\,\mathrm{eV}}$$

nearest-neighbor overlap:

$$\int \phi(\mathbf{r}-\mathbf{r}_B)\,\phi(\mathbf{r}-\mathbf{r}_A)\,d^3\mathbf{r} = s_0$$

often neglected



two bands touch at the corners of the hexagon

Pristine graphene: half-filled bands

2D Dirac electrons

Near the conical points

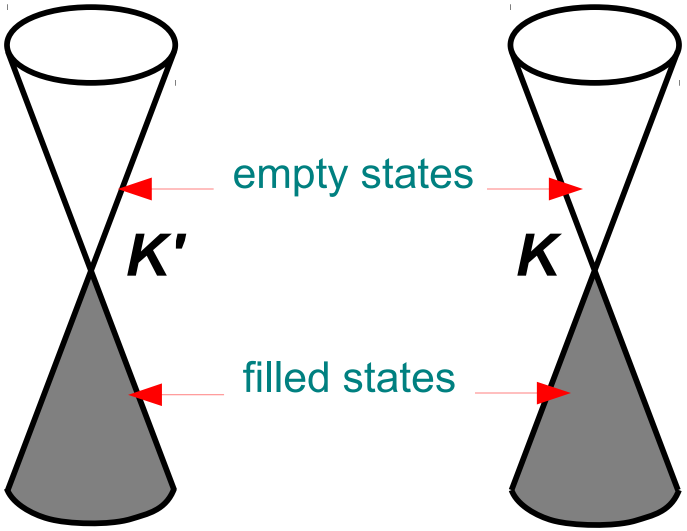
$\mathbf{p} = \mathbf{k} - \mathbf{K}$ or $\mathbf{p} = \mathbf{k} - \mathbf{K}'$

$$\hat{H} = v \begin{pmatrix} 0 & \pm p_x - i p_y \\ \pm p_x + i p_y & 0 \end{pmatrix}$$

$$v = \frac{3}{2}\gamma_0 l_{\mathrm{C-C}} \approx 10^8\,\mathrm{cm/s} \approx 7\,\mathrm{eV}\cdot\mathrm{\AA}$$

from experiments

from the
tight-binding
model

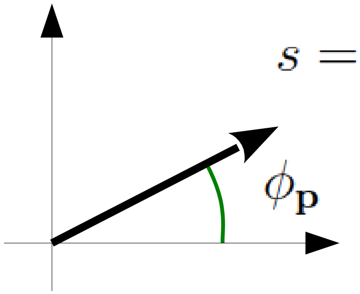


Eigenvectors at the ***K*** point:

$$\psi_{\mathbf{p},s}(\mathbf{r}) = \frac{e^{i\mathbf{p}\mathbf{r}}}{\sqrt{2}} \begin{pmatrix} e^{-i\phi_{\mathbf{p}}} \\ s \end{pmatrix}$$

$$\boxed{\epsilon_{\mathbf{p},s} = s v |\mathbf{p}|}$$

$$s = \pm \begin{cases} \text{conduction band} \\ \text{valence band} \end{cases}$$



$\phi_{\mathbf{p}}$ polar angle of **p**

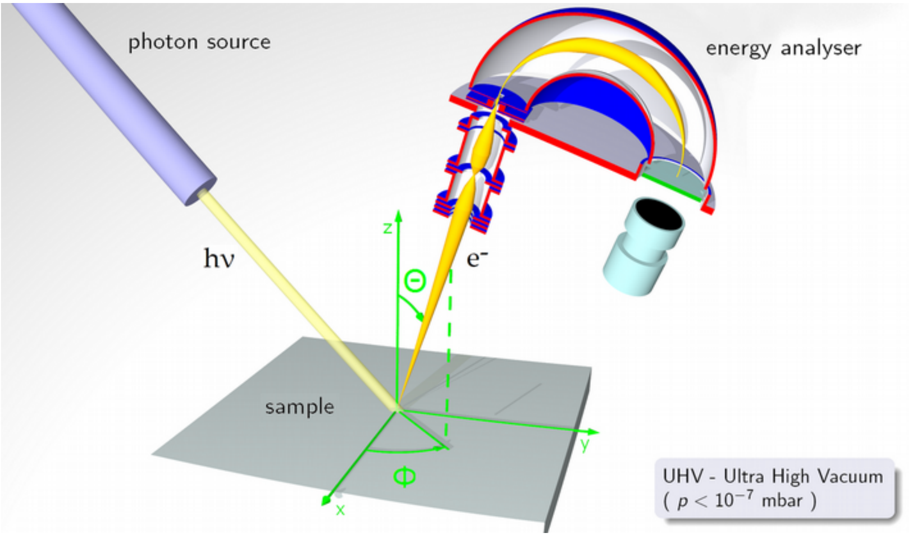
ARPES measurements

Angle-Resolved
Photo-Emission Spectroscopy:

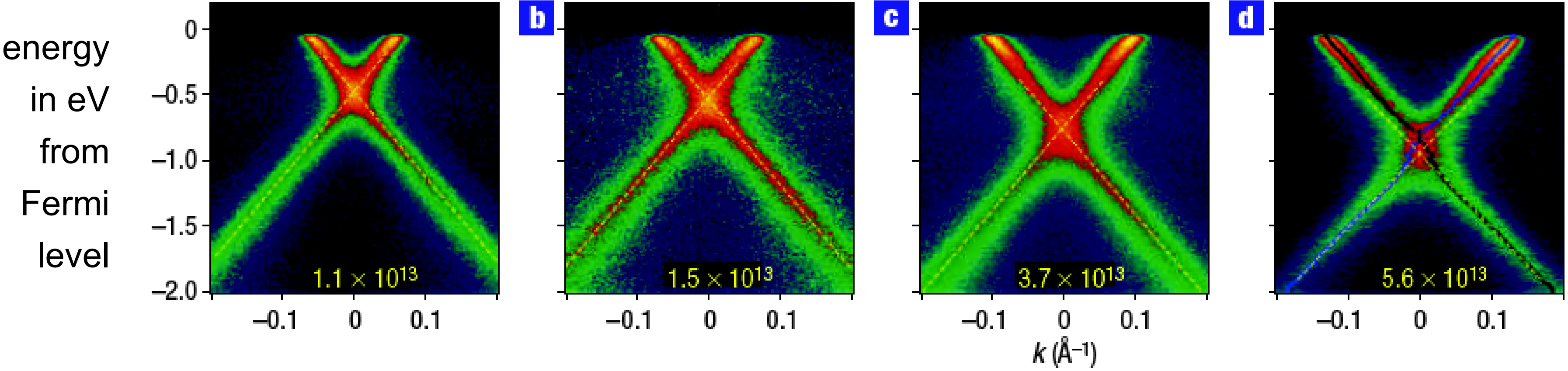
X-ray photon absorbed,
electron emitted
energy-momentum conservation

↓

measure electronic disperson

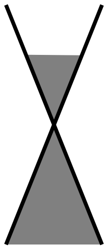


graphene dispersion for different electron densities, from Bostwick *et al*., Nat. Phys. **3**, 36 (2007)

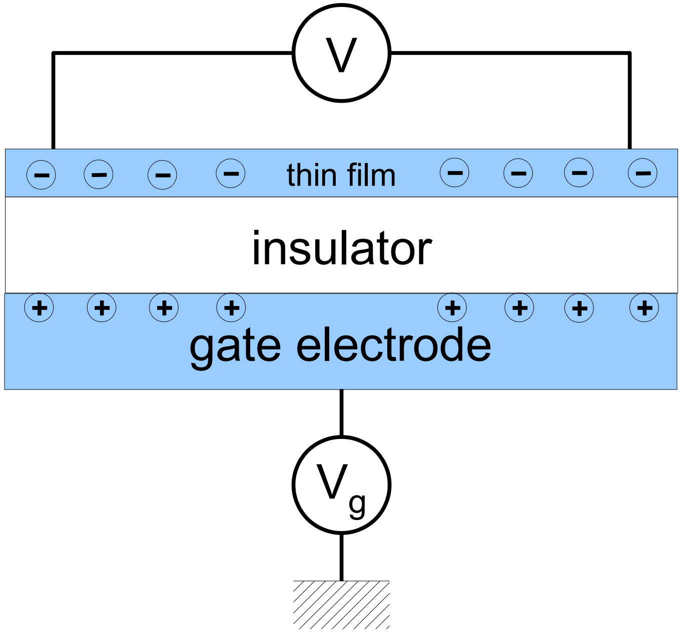


666 22 OCTOBER 2004 VOL 306 SCIENCE www.sciencemag.org

Electric Field Effect in Atomically Thin Carbon Films



add electrons
to the system



V

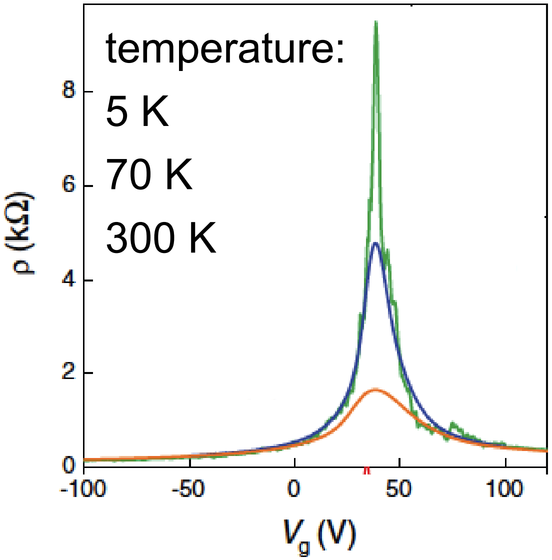
thin film

insulator

gate electrode

V_g

resistivity as a function
of gate voltage



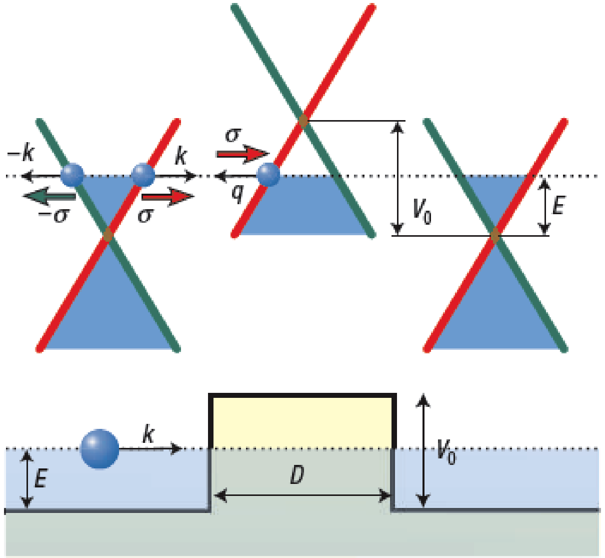
ambipolar field effect

<u>In metals:</u> short screening length
impossibility to obtain very thin films

<u>In semiconductors:</u> one type of carriers dominates

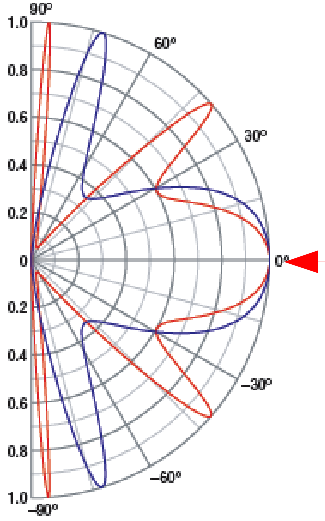
<u>Graphene:</u> layered semimetal

Klein tunneling



Transmission over
a potential barrier
(for two heights):

M. I. Katsnelson,
K. S. Novoselov,
A. K. Geim,
Nature Phys. 2, 620 (2006)



unit transmission
at normal incidence

$$\int \psi^{\dagger}_{-\mathbf{p},s}(\mathbf{r}) \begin{pmatrix} V(\mathbf{r}) & 0 \\ 0 & V(\mathbf{r}) \end{pmatrix} \psi_{\mathbf{p},s}(\mathbf{r})\, d^2\mathbf{r} = 0$$

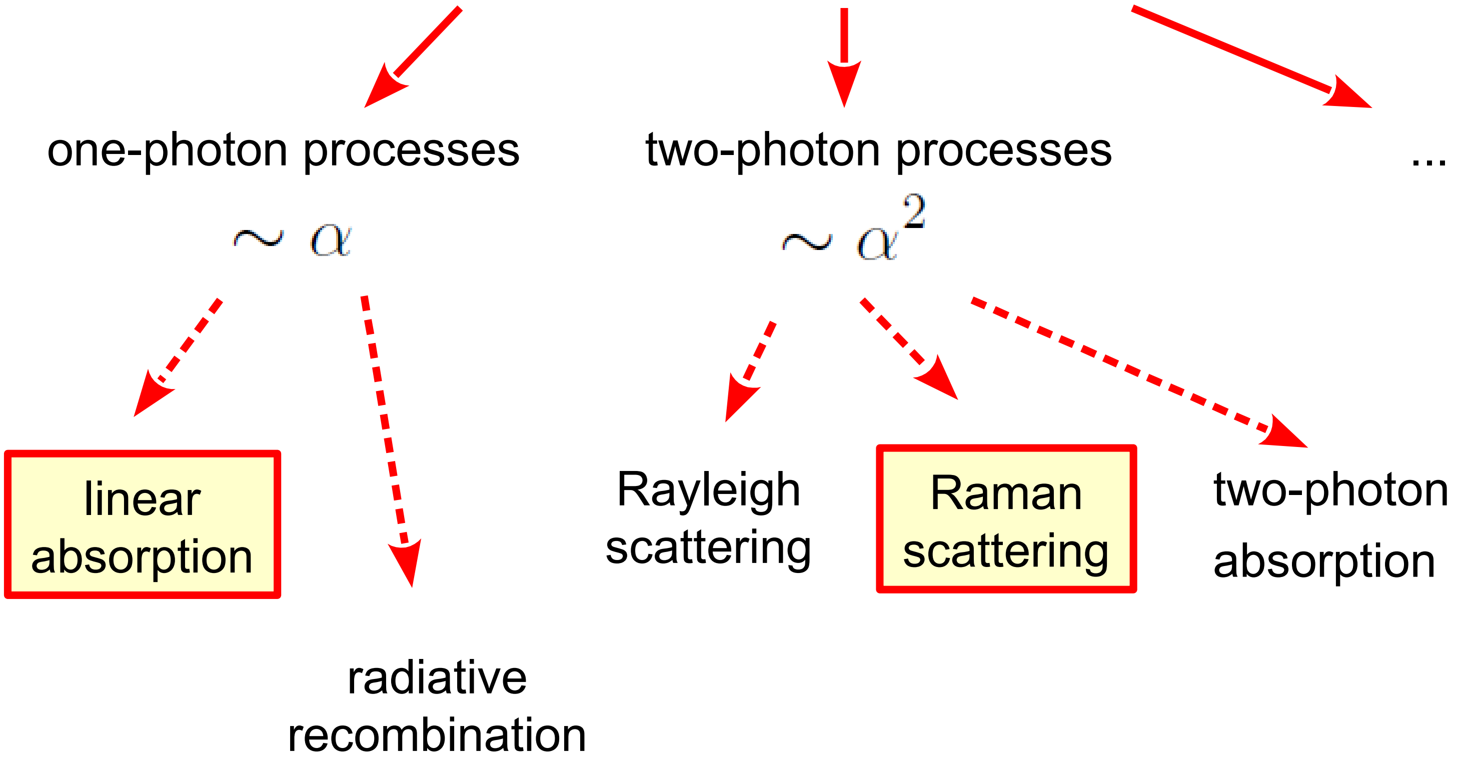
The same potential on
A and B sublattices

No backscattering by a smooth potential
No electrostatic confinement

Interaction of light with a solid

1. All lengths << λ → photon momentum $\frac{\omega}{c} \to 0$

2. Atomic layer → perturbation theory in $\alpha = \frac{e^2}{c} \approx \frac{1}{137}$



one-photon processes

$\sim \alpha$

two-photon processes

$\sim \alpha^2$

...

linear absorption

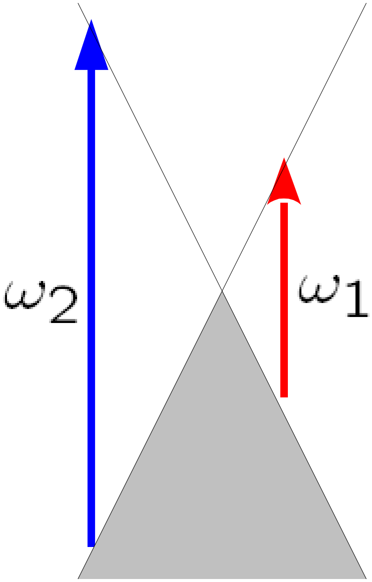
radiative recombination

Rayleigh scattering

Raman scattering

two-photon absorption

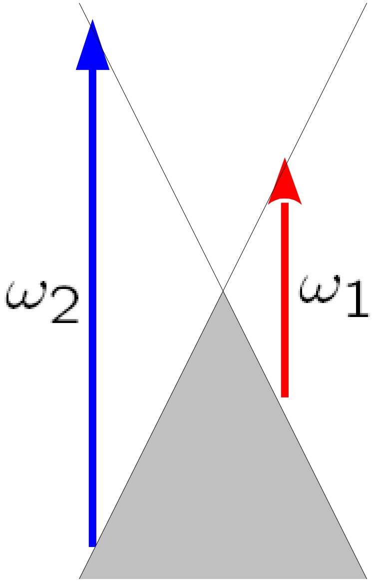
Universal optical absorption

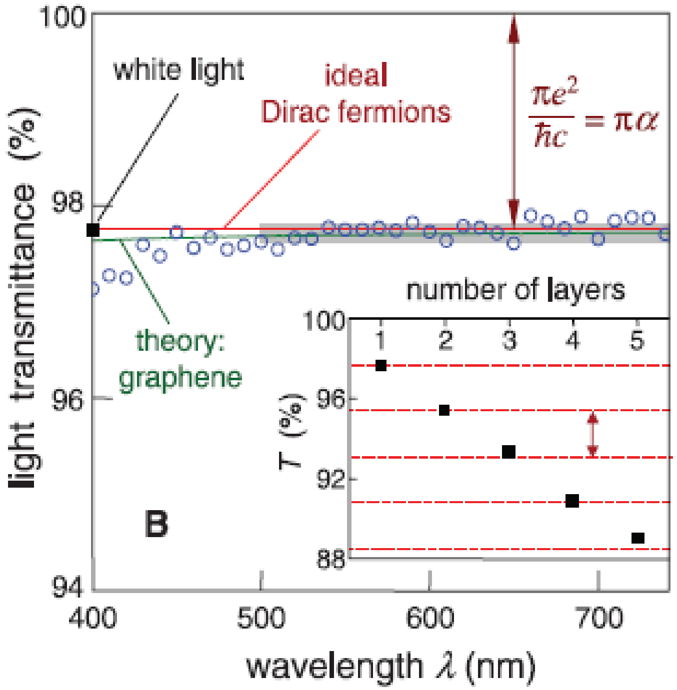


no intrinsic energy scale for
the Dirac spectrum →

$$a(\omega) = (\text{const} \sim 1) \cdot \alpha$$

Universal optical absorption





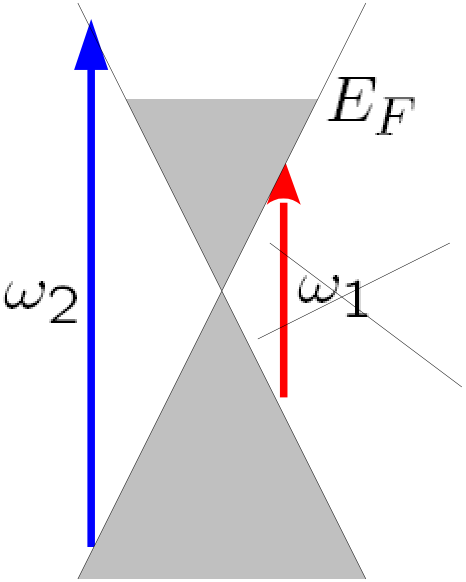
no intrinsic energy scale for the Dirac spectrum →

R.R. Nair *et al*., Science **320**, 1308 (2008)

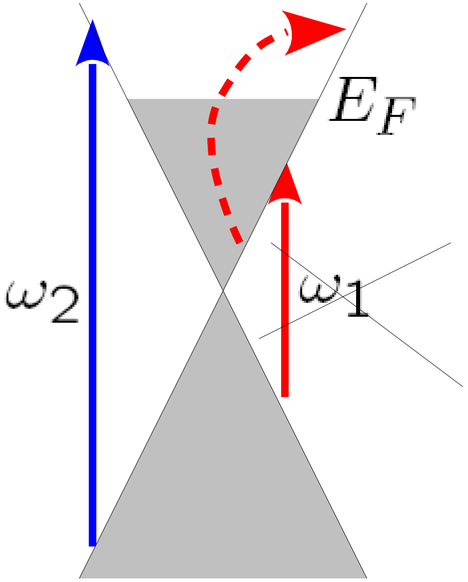
$$a(\omega) = (\mathrm{const} \sim 1) \cdot \alpha$$

$$\mathrm{const} = \pi$$

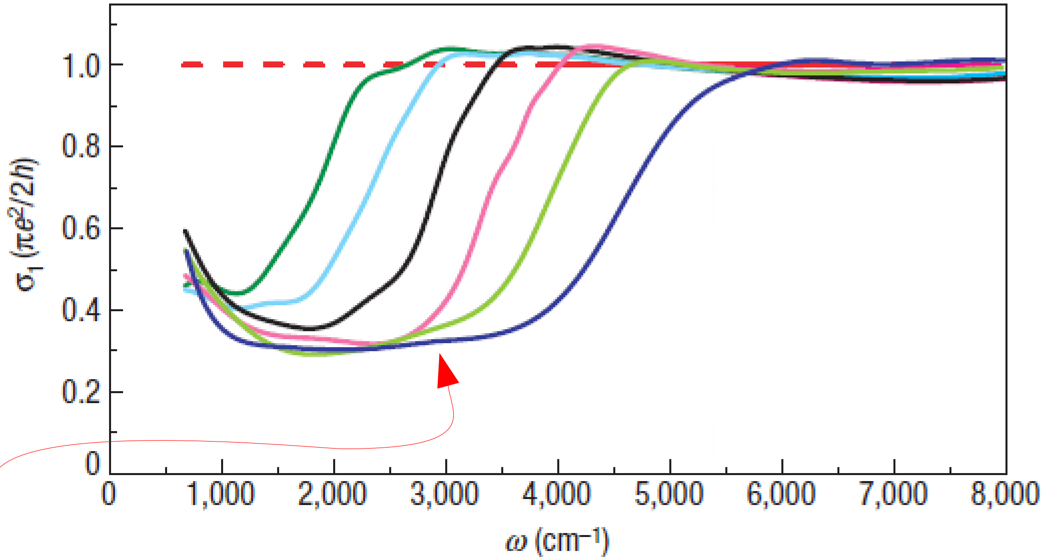
Gate voltage dependence



Gate voltage dependence



$$a(\omega) = \frac{4\pi}{c}\,\mathrm{Re}\,\sigma(\omega)$$



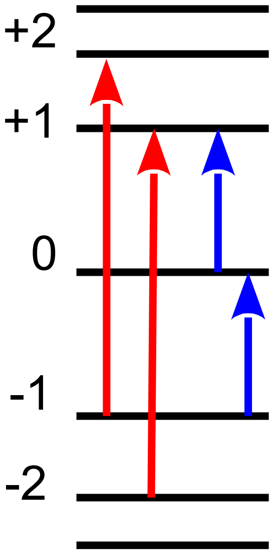
Z. Q. Li *et al*., Nature Phys. **4**, 532 (2008)

Residual conductivity: broken momentum conservation

– Coulomb impurities [T. Stauber *et al*., PRB **78**, 085418 (2008);
P. Yuan *et al*., PRB 84, 195418 (2011)]

– electron-electron scattering [A. G. Grushin *et al*., PRB **80**, 155417 (2009)]

Landau level spectroscopy



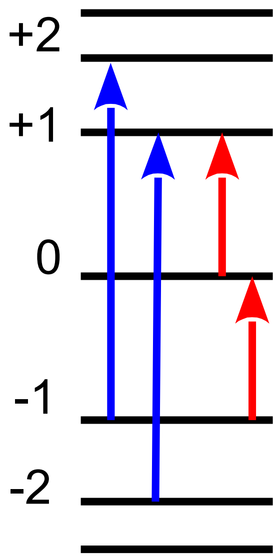
dipolar selection rule:

$$\Delta|n| = \pm 1$$

$$\epsilon_n = \pm\sqrt{2ev^2B|n|}$$

experimental access to the Dirac velocity

Landau level spectroscopy



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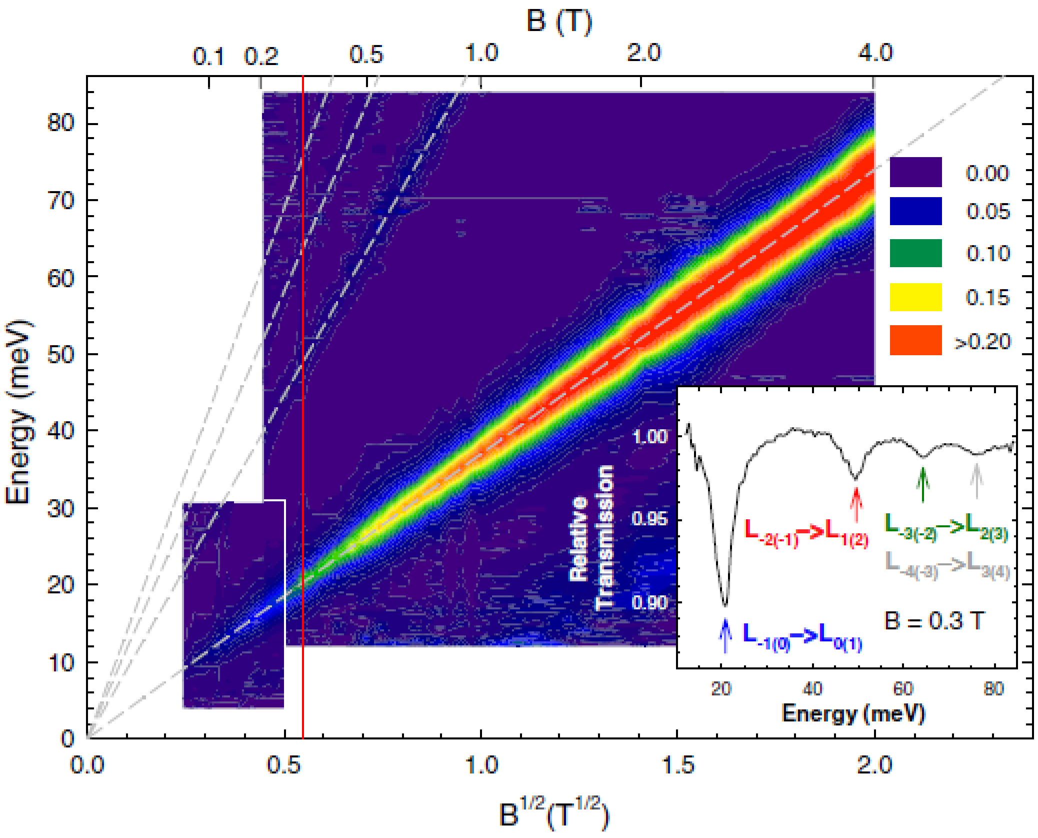
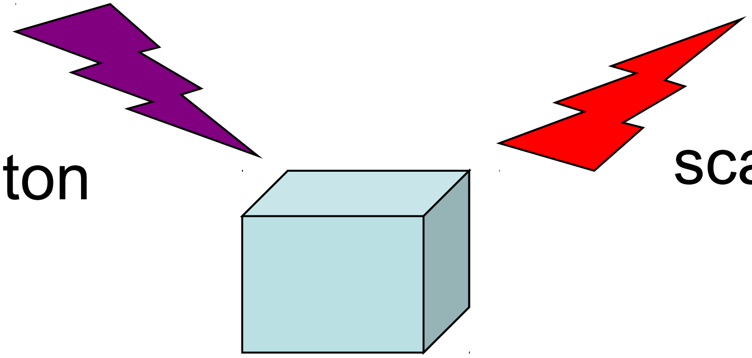


M. Orlita *et al.*, PRL **101**, 267601 (2008)

Raman scattering

L. I. Mandelshtam and G. S. Landsberg, Zeitschrift für Physik **50**, 169 (1928)

C. V. Raman and K. S. Krishnan, Nature **121**, 501 (1928)



ω_{in}

incident photon

ω_{out}

scattered photon

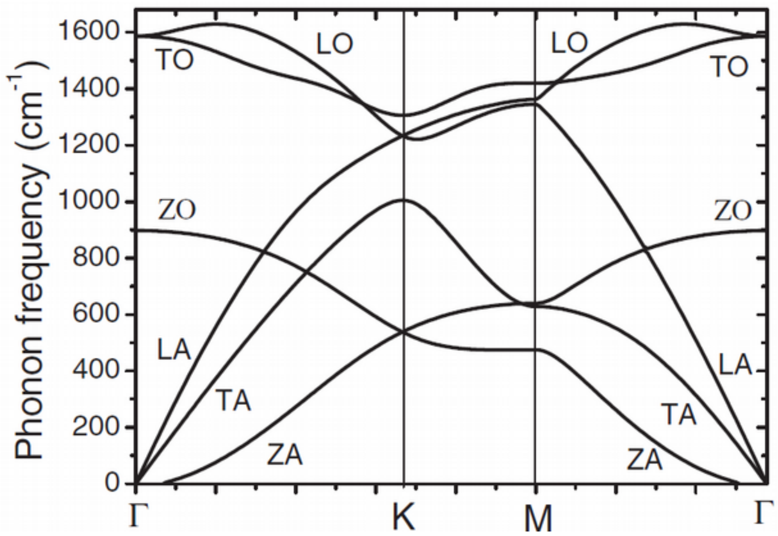
sample

$\omega_{\text{out}} = \omega_{\text{in}}$ – Rayleigh scattering (elastic)

$\omega_{\text{out}} \neq \omega_{\text{in}}$ – Raman scattering

Some excitation is left in the system

Phonons in graphene

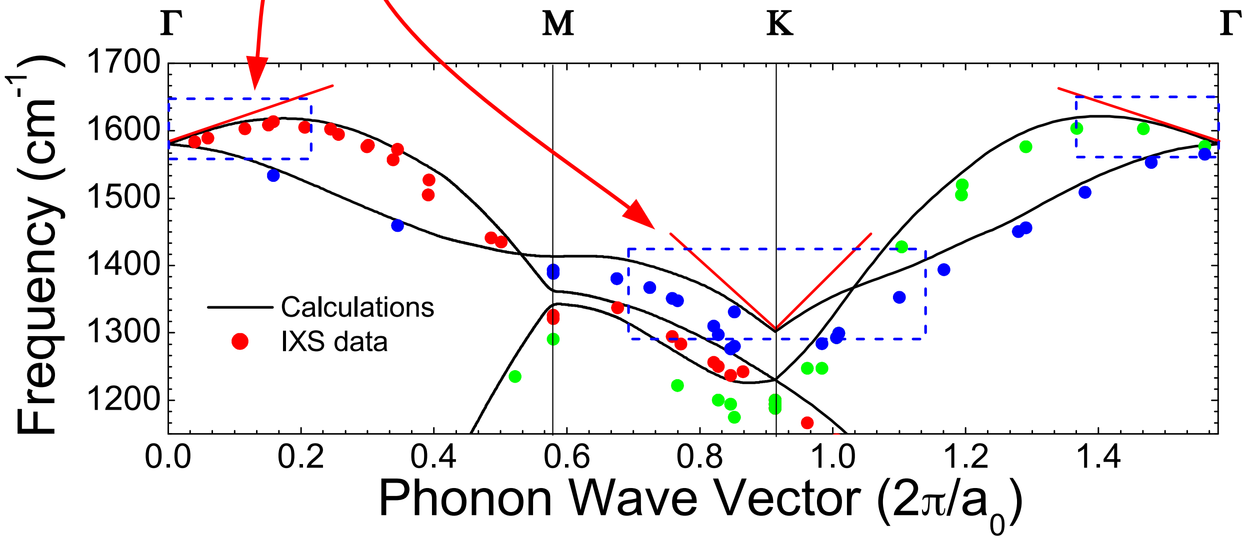


Mechanical phonon dispersion from coupling to deep electronic bands

from Yan, Ruan and Chow, PRB **77**, 125401 (2008)

Kohn anomalies from coupling to electrons near the Fermi level:

sensitive to doping and magnetic field



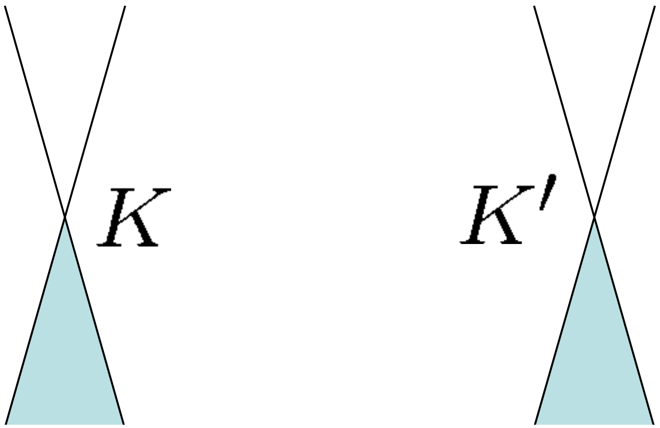
Calculation:
Piscanec *et al.*,
PRL **93**, 185503 (2004)

Inelastic X-ray scattering:
Maultzsch *et al.*,
PRL **92**, 075501 (2004)

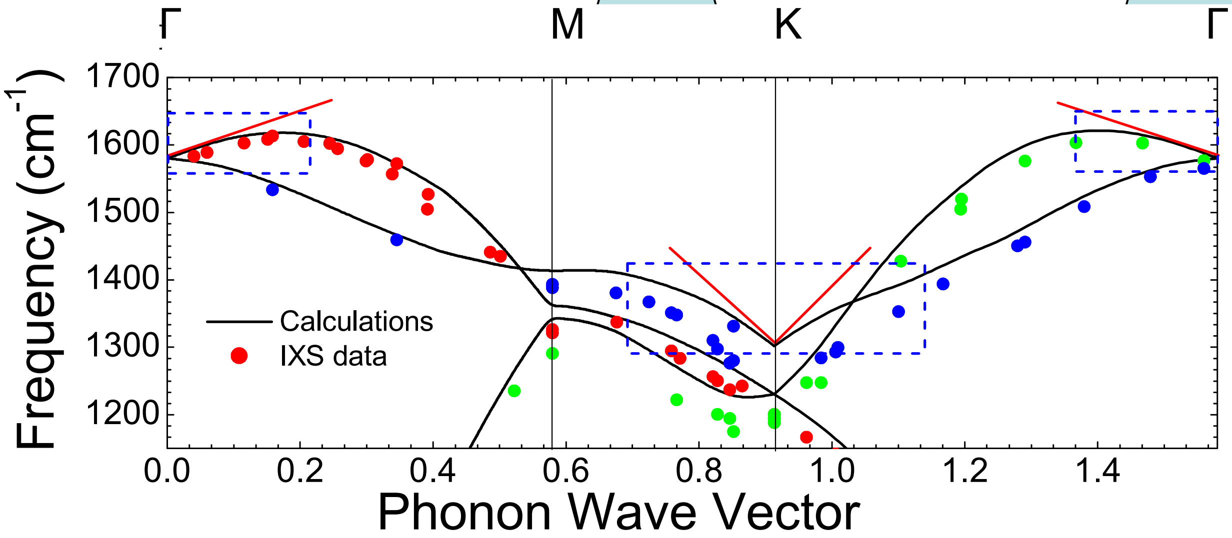
Raman scattering: photons ↔ electrons ↔ phonons

intermediate states

Electron spectrum:
two Dirac cones



optical
phonon
spectrum



DFT calculation: Piscanec *et al.*, PRL **93**, 185503 (2004)

Inelastic X-ray scatt.: Maultzsch *et al*., PRL **92**, 075501 (2004)

Raman scattering: photons ↔ electrons ↔ phonons

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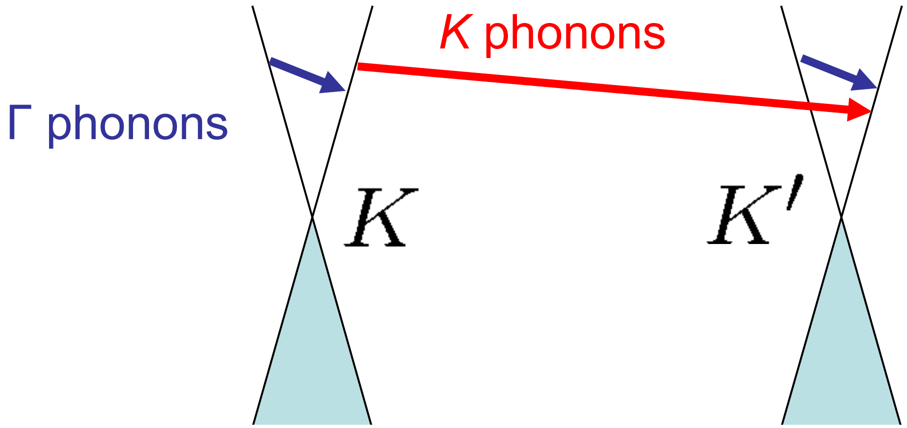

Electron spectrum:
two Dirac cones

optical
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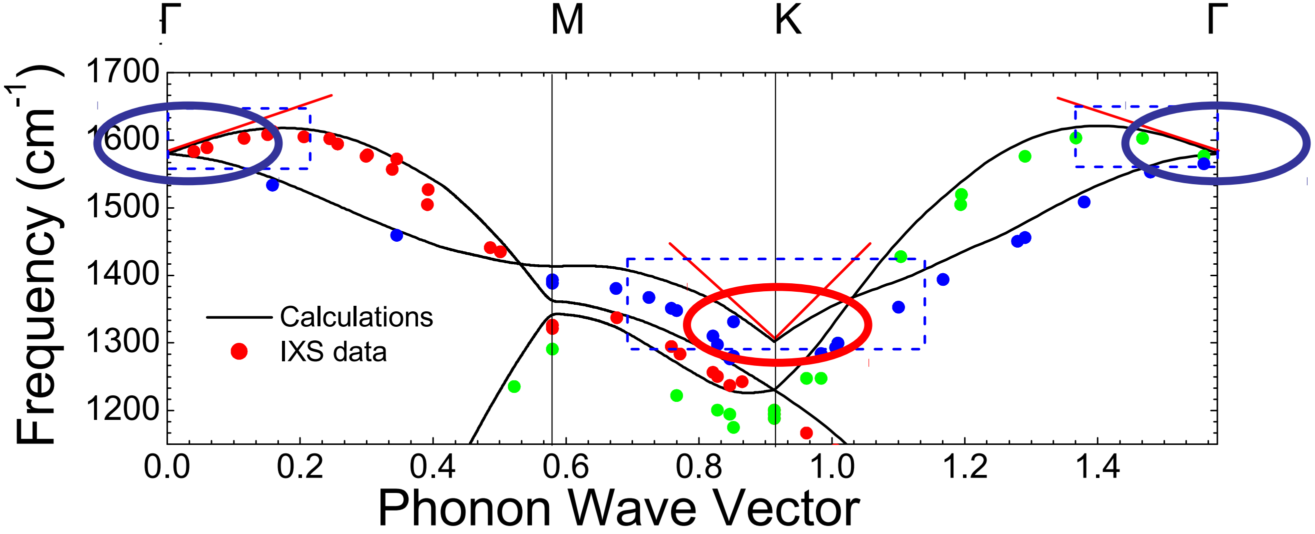
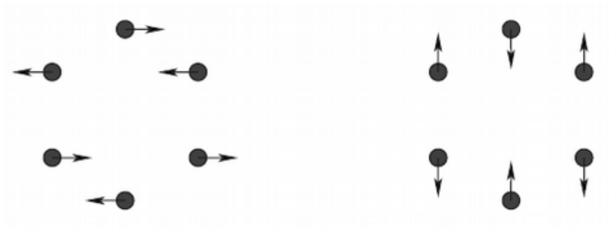


DFT calculation: Piscanec *et al.*, PRL **93**, 185503 (2004)

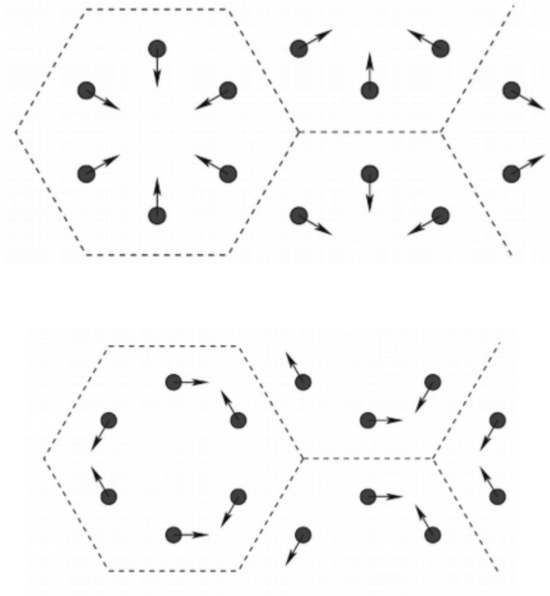
Inelastic X-ray scatt.: Maultzsch *et al.*, PRL **92**, 075501 (2004)

Phonons in graphene

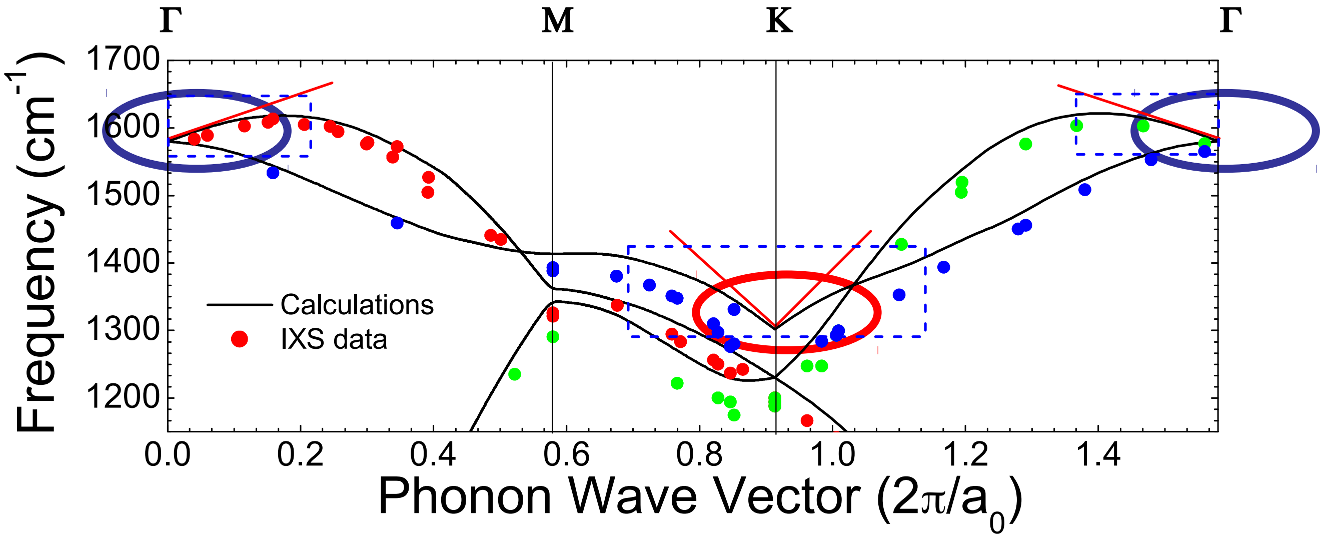
E_2 optical phonons @ Γ



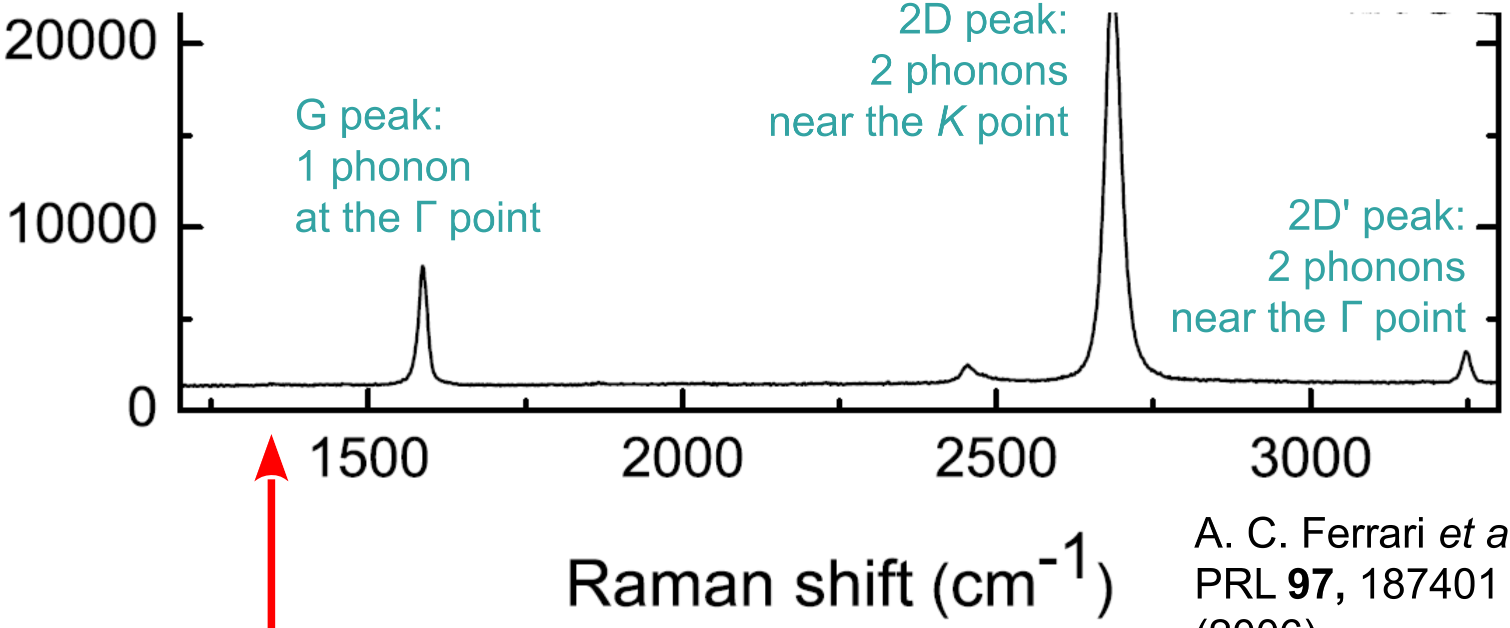
real linear
combinations
of A_1 phonons
@ *K* and *K'*



optical
phonon
spectrum



Raman spectrum of graphene



G peak:
1 phonon
at the Γ point

2D peak:
2 phonons
near the *K* point

2D' peak:
2 phonons
near the Γ point

A. C. Ferrari *et al.*
PRL **97,** 187401
(2006)

no *D* peak (1 phonon @ *K*):
momentum conservation,
no impurities

Raman scattering probability

$$\sum_{\vec{q}_1+\ldots+\vec{q}_n=0} |\mathcal{M}(\vec{q}_1,\ldots,\vec{q}_n)|^2 2\pi\delta(\omega+\omega_{\vec{q}_1}+\ldots+\omega_{\vec{q}_n}-\omega_{in})$$

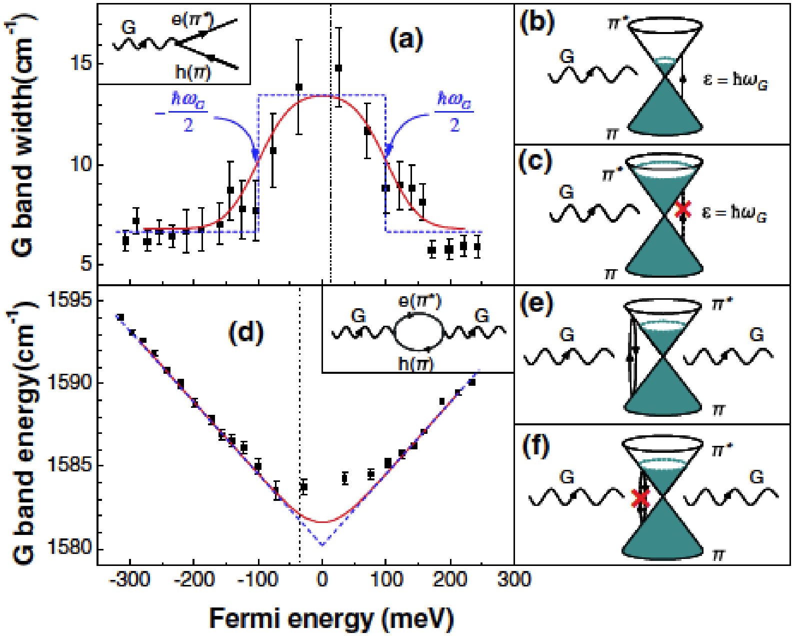
information about the final states
(the phonon spectrum)

most suitable: G peak

Measure the shift of the G peak as a function of

- gate voltage
- magnetic field
- strain
- impurity concentration
- ...

Effect of charging on the phonon frequency, probed by Raman scattering



$$\delta\omega_\Gamma = \frac{\lambda_\Gamma}{2\pi}\left(|E_F| + \frac{\omega_\Gamma}{4}\ln\frac{2E_F-\omega_\Gamma}{2E_F+\omega_\Gamma}\right)$$

shift,
proportional
to the Fermi
energy

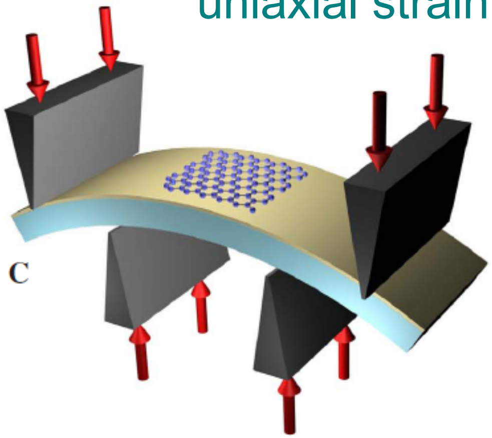
imaginary part
gives phonon
damping

T. Ando, J. Phys. Soc. Jpn. **75**, 124701 (2006)

J. Yan *et al*., PRL **98**, 166802 (2007)
see also S. Pisana *et al*., Nature Materials **6**, 198 (2007)

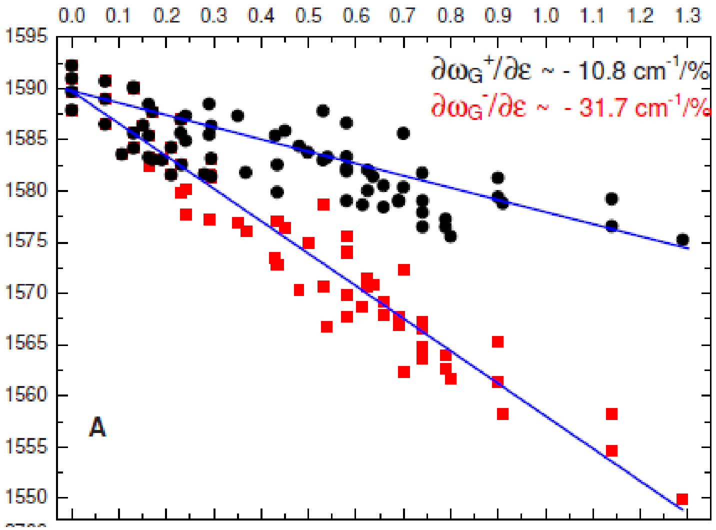
Effect of strain on the G peak

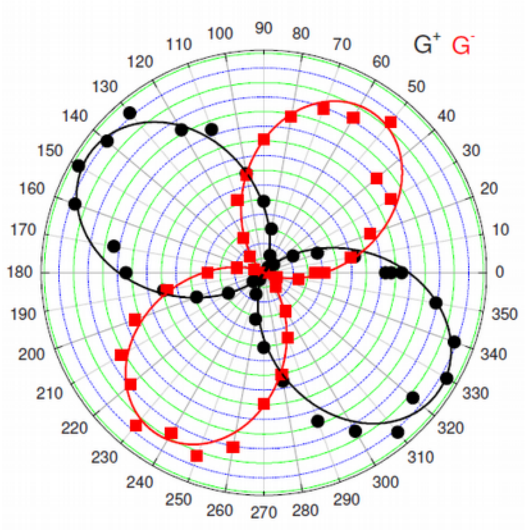
uniaxial strain



Strain → anisotropy:

G peak shift and splitting





Polarization dependence of the two components:

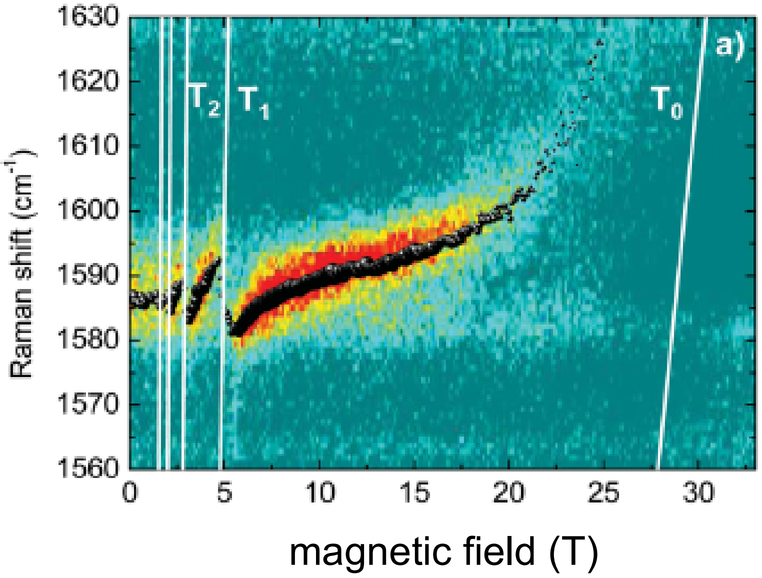
$$I_{G^-} \propto \sin^2(\theta_{\text{in}} + \theta_{\text{out}} + 3\varphi_s), \quad I_{G^+} \propto \cos^2(\theta_{\text{in}} + \theta_{\text{out}} + 3\varphi_s)$$

possibility to determine the absolute crystal orientation!

T. M. G. Mohiuddin *et al.*, PRB **79**, 205433 (2009)

Effect of a magnetic field on the phonon frequency, probed by Raman scattering

C. Faugeras *et al.*, PRL **103**, 186803 (2009)



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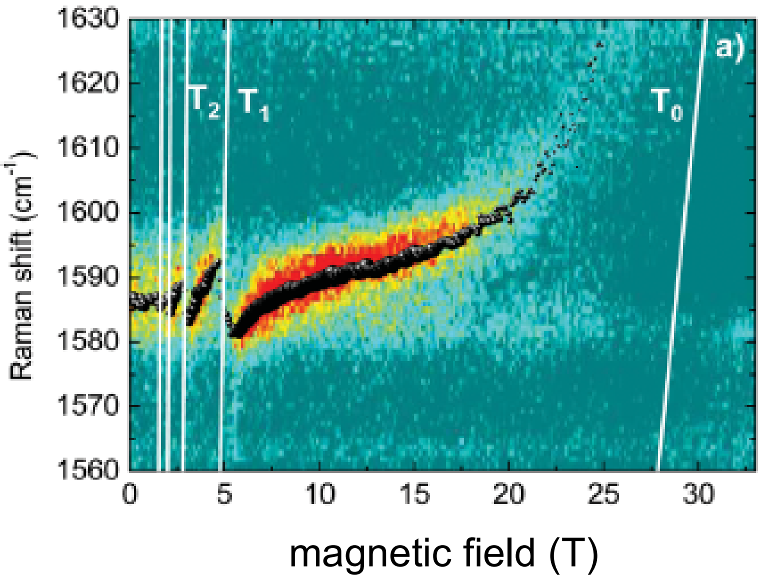
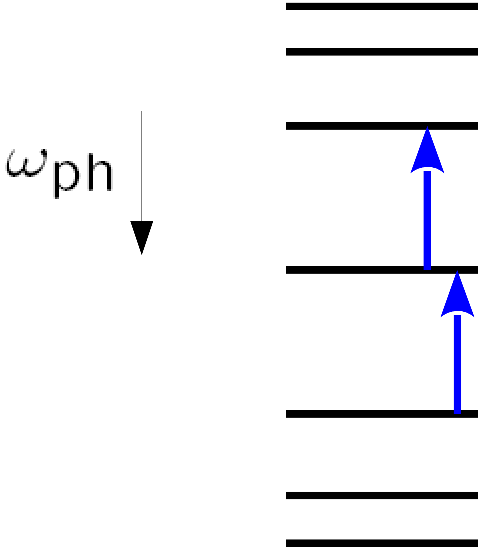




avoided crossing between the phonon and an electronic inter-Landau-level excitation

T. Ando, J. Phys. Soc. Jpn. **76**, 024712 (2007)
M. O. Görbig *et al.*, PRL **99**, 087402 (2007)

Note: the splitting depends on the electronic occupations!

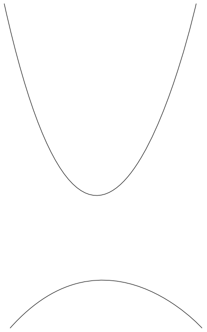
Raman scattering probability

$$\sum_{\vec{q}_1+\ldots+\vec{q}_n=0} |\mathcal{M}(\vec{q}_1,\ldots,\vec{q}_n)|^2 2\pi\delta(\omega+\omega_{\vec{q}_1}+\ldots+\omega_{\vec{q}_n}-\omega_{in})$$

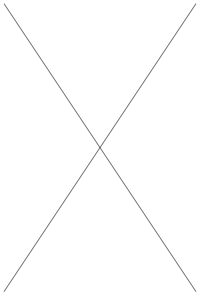
information about
the intermediate states
(electrons and holes)

most suitable: 2D peak

Theory for semiconductors:
1960-70-ies



Dirac spectrum: *e*-*h* symmetry



additional symmetry
brings additional physics

But: $m_e = m_h$ in InBr, InI

I. G. Lang *et al*., Sov. Phys. Solid State **32**, 2000 (1990)

One-phonon Raman scattering

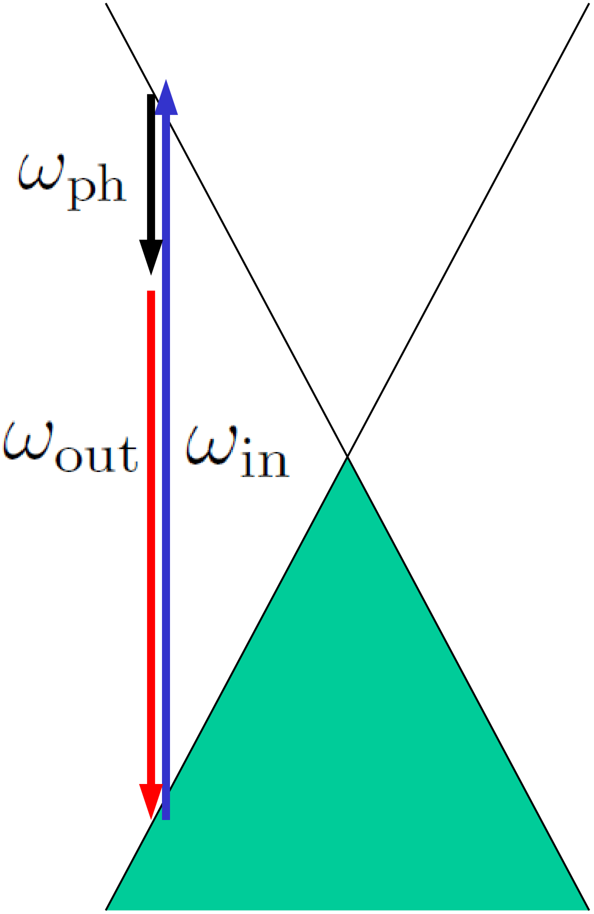
Photon wave vector $\frac{\omega_{in}}{c} \to 0$

No disorder → momentum conservation

emit phonon with $q = 0$

$$\mathcal{M} = \sum_{1,2} \frac{\langle f|\hat{H}_{e-light}|2\rangle\langle 2|\hat{H}_{e-ph}|1\rangle\langle 1|\hat{H}_{e-light}|i\rangle}{(\omega_{in} - E_1 + 2i\gamma)(\omega_{in} - E_2 + 2i\gamma)}$$

At least one virtual state:
energy denominator ~ ω_{ph}



Numerator in the Dirac approximation:

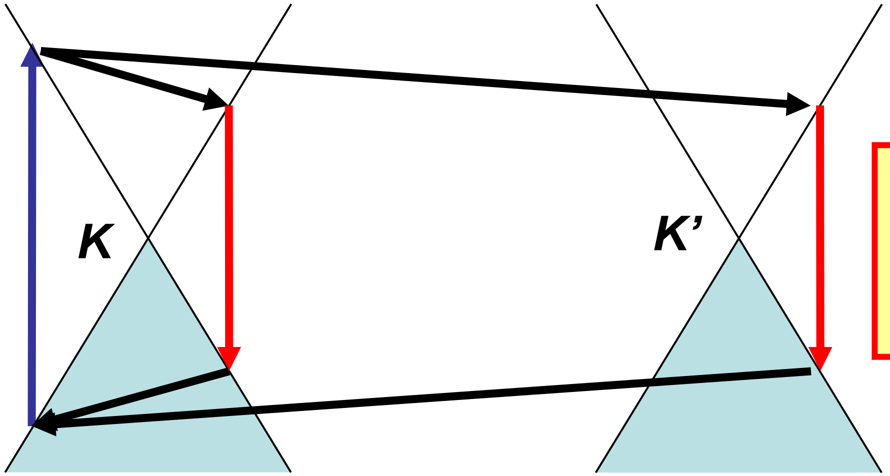
$$\hat{H}_{e-light} \propto \sigma_x, \sigma_y, \; \hat{H}_{e-ph} \propto \sigma_x, \sigma_y$$

angular momentum ±1 ⇨ $\boxed{\mathcal{M} = 0}$

Trigonal warping must be included

+1 +1 + 1 = +3 = 0 – cross-circular polarization

Two-phonon Raman scattering: $\vec{q}, -\vec{q}$



Full resonance:
all intermediate states are real → the amplitude is determined by *iγ*

$$\mathcal{M} = \sum_{1,2,3} \frac{\langle f|\hat{H}_{e-light}|3\rangle\langle 3|\hat{H}_{e-ph}|2\rangle\langle 2|\hat{H}_{e-ph}|1\rangle\langle 1|\hat{H}_{e-light}|i\rangle}{(\omega_{in} - E_1 + 2i\gamma)(\omega_{in} - E_2 + 2i\gamma)(\omega_{in} - E_3 + 2i\gamma)}$$

Dimensionless quantum efficiency of the 2D and 2D' peaks:

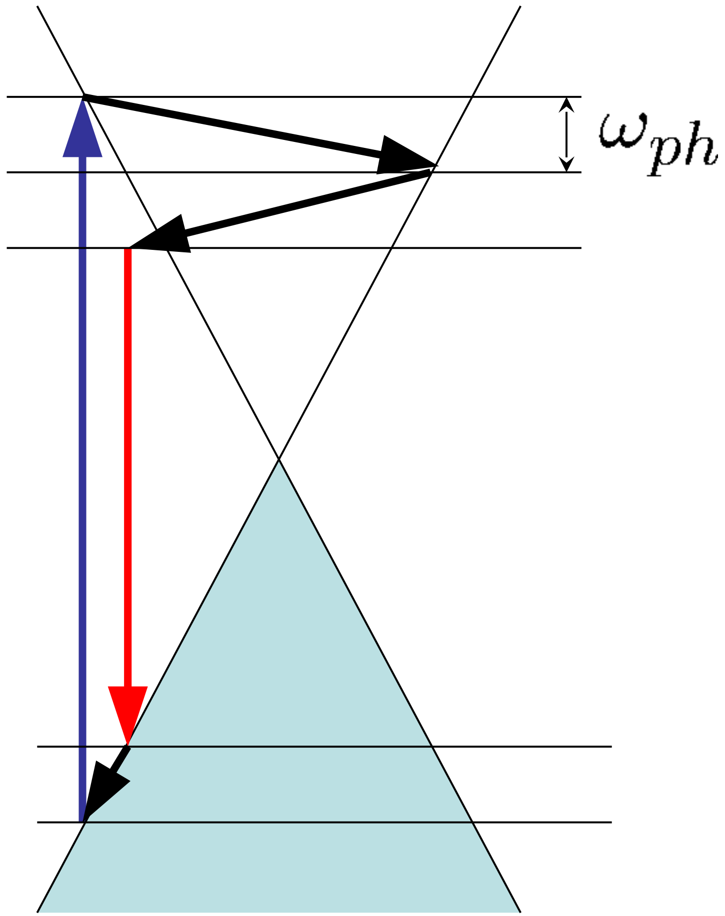
$$\boxed{I_{2D} = \frac{\alpha^2\lambda_K^2}{48}\frac{\omega_{in}^2}{c^2}\frac{v^2}{\gamma^2}} \quad \boxed{I_{2D'} = \frac{\alpha^2\lambda_\Gamma^2}{24}\frac{\omega_{in}^2}{c^2}\frac{v^2}{\gamma^2}} \quad \lambda_\mu = \frac{F_\mu^2}{\rho v^2 \omega_\mu}$$

dimensionless
e-ph coupling
strength

↑ mass density of the crystal

D. M. Basko, PRB **78**, 125418 (2008)

Three-phonon Raman scattering: $\vec{q}_1 + \vec{q}_2 + \vec{q}_3 = 0$



1. Odd number of phonons:
at least one virtual state

2. Even number of phonons:
all denominators can vanish
simultaneously

These are the consequences of the electron-hole symmetry

I. G. Lang *et al.*, Sov. Phys. Solid State **32**, 2000 (1990)

Raman matrix element in the real space

$$\mathcal{M}=\int\frac{d\epsilon}{2\pi}\int\prod_i d\vec{r}_i\ldots G(\vec{r}_2-\vec{r}_1,\epsilon-\omega_{\rm ph})\,e^{i\vec{q}_1\vec{r}_1}\,G(\vec{r}_1-\vec{r}_0,\epsilon)\ldots$$

hierarchy of energy scales:

ϵ ~ 1 eV

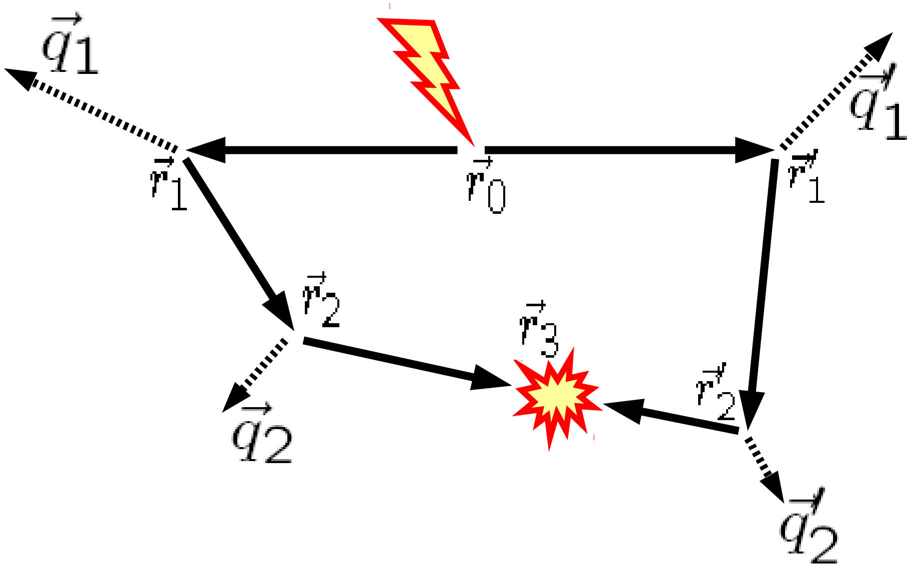
$\omega_{\rm ph}$ 170 meV

γ ~ 20—30 meV

$$v/\epsilon \ll v/\omega_{\rm ph}, v/\gamma$$

electron wavelength

spatial extent of the process
for even number of phonons or
for odd number of phonons
from the uncertainty principle



Green’s functions admit a quasiclassical representation

Raman matrix element in the real space

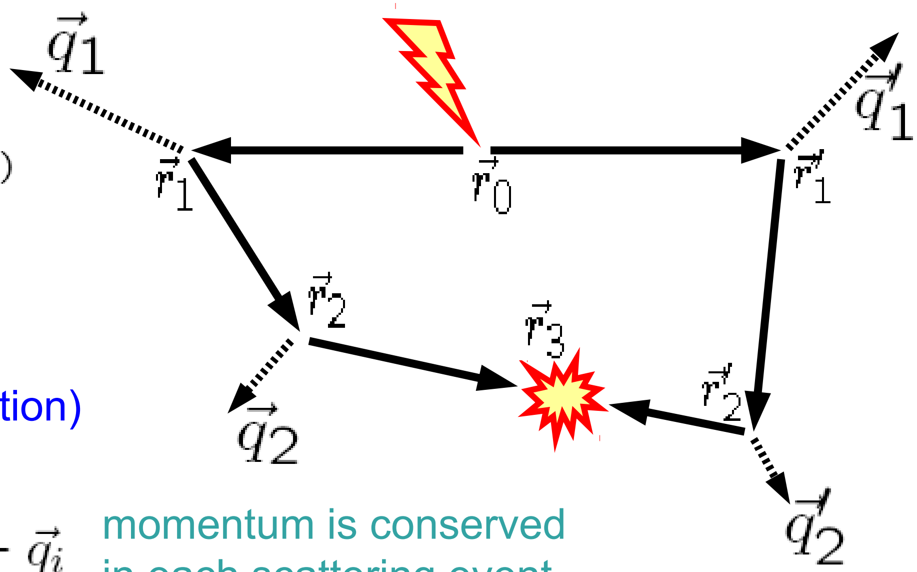
$$\mathcal{M} = \int \frac{d\epsilon}{2\pi} \int \prod_i d\vec{r}_i \ldots G(\vec{r}_2-\vec{r}_1, \epsilon-\omega_{\text{ph}})\, e^{i\vec{q}_1\vec{r}_1}\, G(\vec{r}_1-\vec{r}_0, \epsilon) \ldots$$

quasiclassical representation:

$$G(\vec{r}, \vec{r}', \epsilon) = A(\vec{r}, \vec{r}', \epsilon)\, e^{iS(\vec{r}, \vec{r}', \epsilon)}$$

slow amplitude

fast eikonal (classical action)



$$\frac{\partial S}{\partial \vec{r}} = \vec{p} \quad \Rightarrow \quad \vec{p}_i = \vec{p}_{i-1} + \vec{q}_i$$

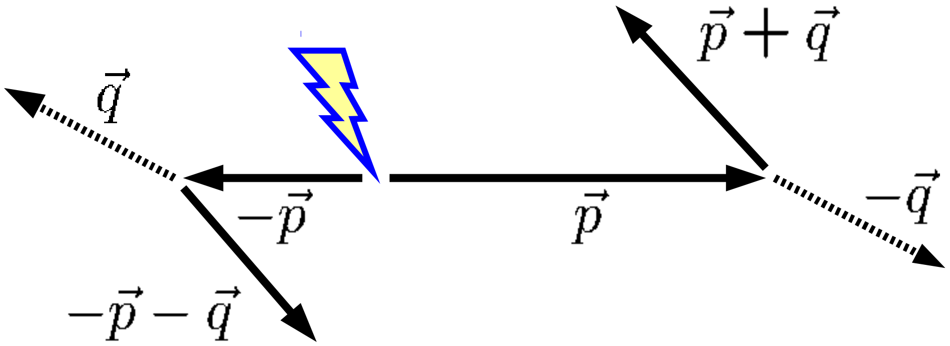
momentum is conserved in each scattering event

$$\frac{\partial S}{\partial \epsilon} = t \quad \Rightarrow \quad t_1 + t_2 + t_3 = t'_1 + t'_2 + t'_3$$

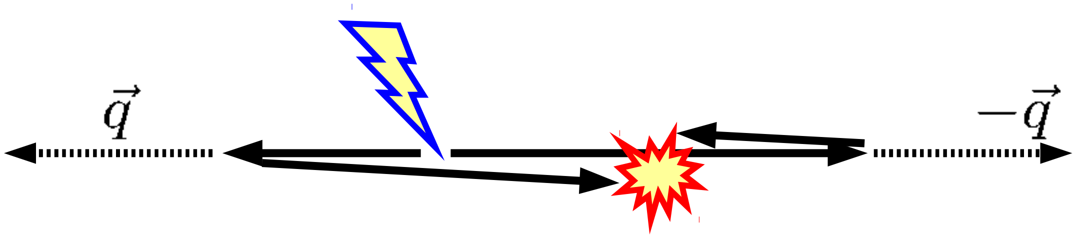
the electron and the hole travel for the same amount of time

The quasiclassical picture works both for real and virtual processes

Two-phonon process: backscattering



The process looks allowed
by momentum conservation
but the electron and the hole
cannot meet in space!



the angular spread is limited
by the quantum uncertainty

$$|\varphi-\pi|\sim\sqrt{\gamma/\omega_{in}}$$

The phonon momentum is fixed:

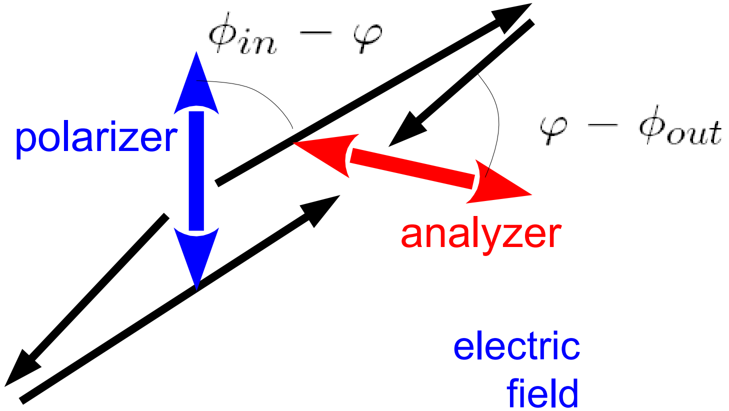
$$|\vec{q}|=|\vec{p}_0|+|\vec{p}_1|=\frac{\omega_{in}+\omega_{out}}{2v}$$

The two-phonon peak
(a) is narrow ($\omega_{\vec{q}}+\omega_{-\vec{q}}$ is not arbitrary)
(b) depends on the excitation frequency

$$\frac{d\omega_{2D}}{d\omega_{\mathrm{in}}}=\frac{2v_{\mathrm{ph}}}{v}\approx 100\frac{\mathrm{cm}^{-1}}{\mathrm{eV}}$$

Vidano *et al.,* Solid State. Comm. **39**, 341 (1981)

Two-phonon process: polarization dependence



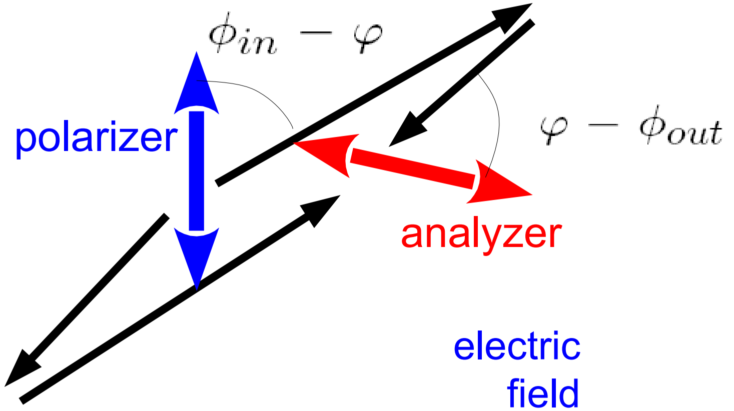
electric field

electron-photon matrix element $\propto [\vec{E} \times \vec{p}]_z$

electron momentum

$$
\begin{aligned}
I_{2D} &\propto \int \sin^2(\phi_{in} - \varphi)\sin^2(\varphi - \phi_{out})\,d\varphi \\
&\propto \frac{1}{3} + \frac{2}{3}\cos^2(\phi_{in} - \phi_{out})
\end{aligned}
$$

Two-phonon process: polarization dependence



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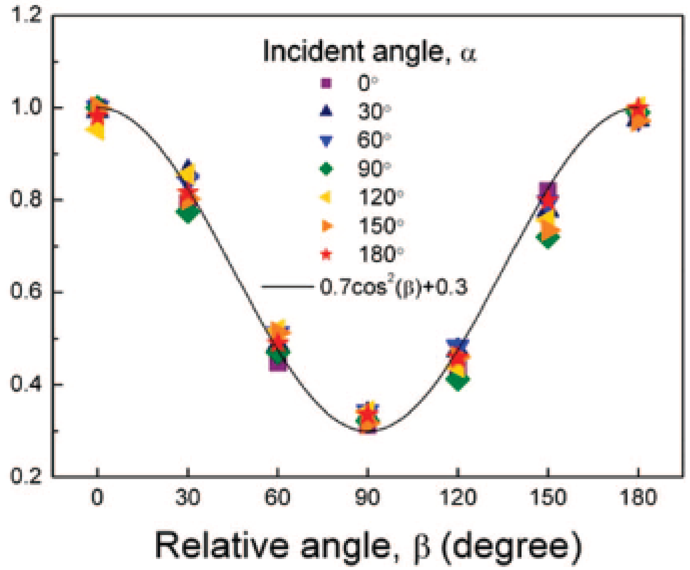
electron momentum

$$I_{2D} \propto \int \sin^2(\phi_{in} - \varphi)\sin^2(\varphi - \phi_{out})\, d\varphi$$
$$\propto \frac{1}{3} + \frac{2}{3}\cos^2(\phi_{in} - \phi_{out})$$

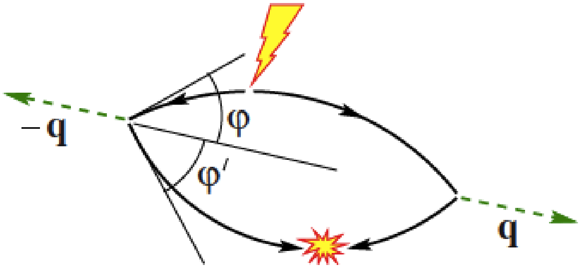


D. Yoon *et al*., Nano Lett. **8**, 4270 (2008)

Strong dependence on the **relative** angle between the polarizer and the analyzer

The scattered photon remembers about the polarization of the incident photon

Two-phonon process: magnetic field dependence



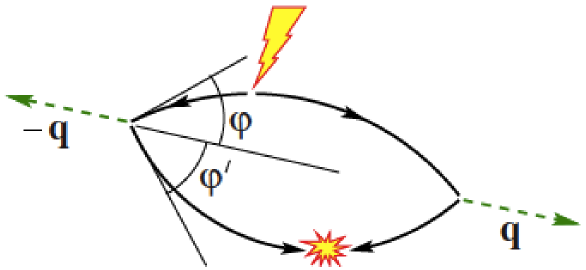
Electronic trajectories are circles:

$|\vec{q}| \approx 2|\vec{p}|\cos\varphi$ instead of $|\vec{q}| \approx 2|\vec{p}|$

$\frac{d\omega_{\vec{q}}}{dq} > 0$ → red shift of the peak

random length $\sim v/\gamma$ → broadening

Two-phonon process: magnetic field dependence



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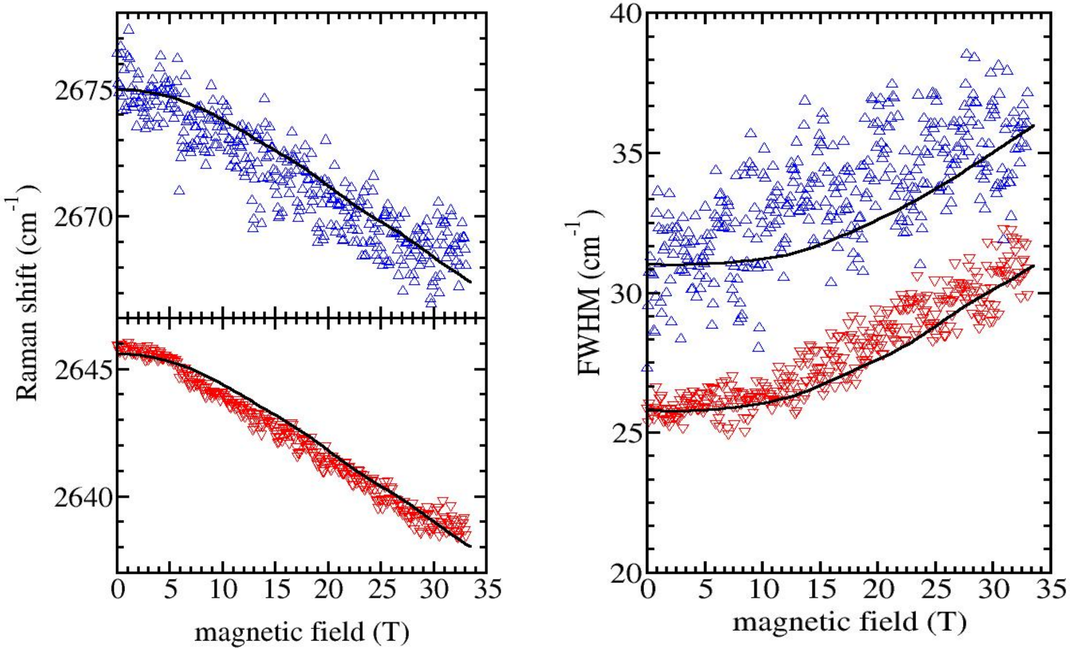


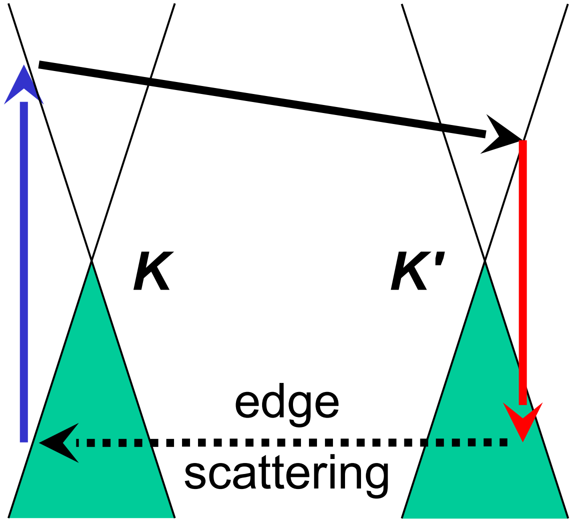
C. Faugeras *et al.*,
Phys. Rev. B **81**,155436 (2010)

The fit allows to extract

$\gamma = 27$ meV

for $\epsilon = 0.85$ eV

Edge-assisted one-phonon Raman scattering

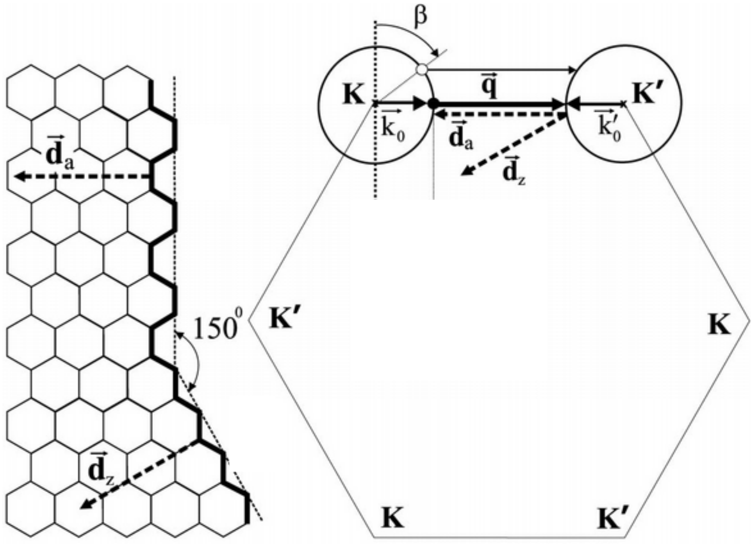


Energy mismatch ~ ω_{ph}= 0.17 eV

↕ *uncertainty principle*

Lifetime of the virtual state ~ 1/ω_{ph}

Electron energy $\epsilon \sim 1\text{ eV} \gg \omega_{ph}$



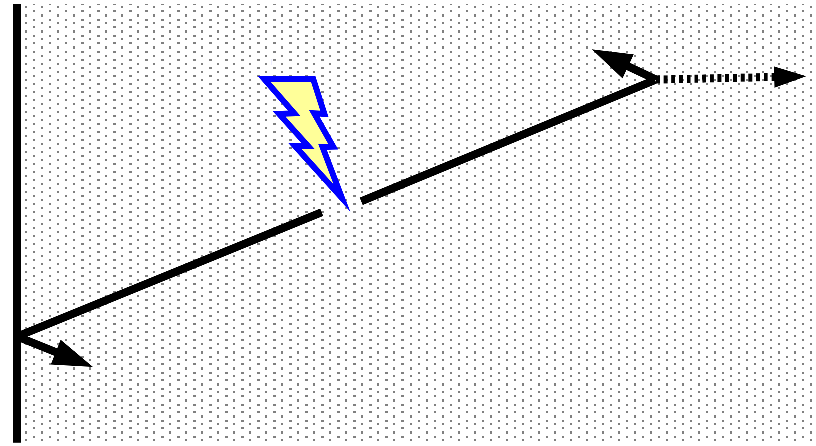
An ideal zigzag edge cannot scatter an electron from ***K*** to ***K'***

↓

No D peak

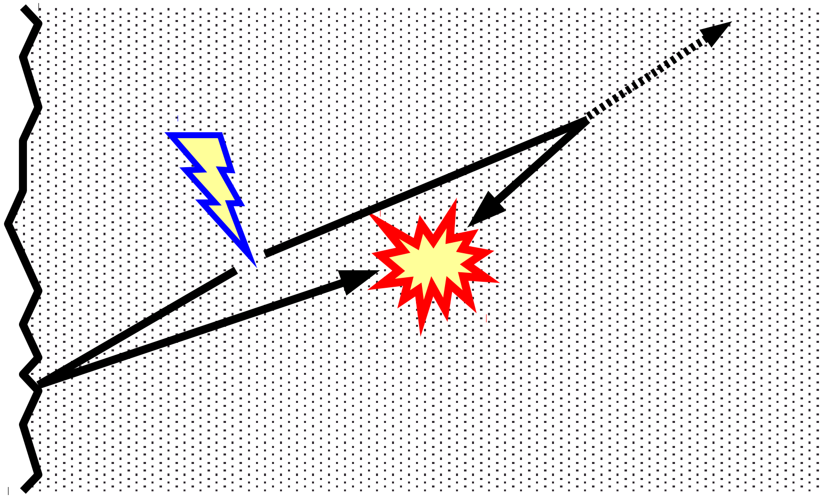
Cançado *et al.*, PRL **93,** 247401 (2004)

Backscattering condition



perfect edge, oblique incidence:

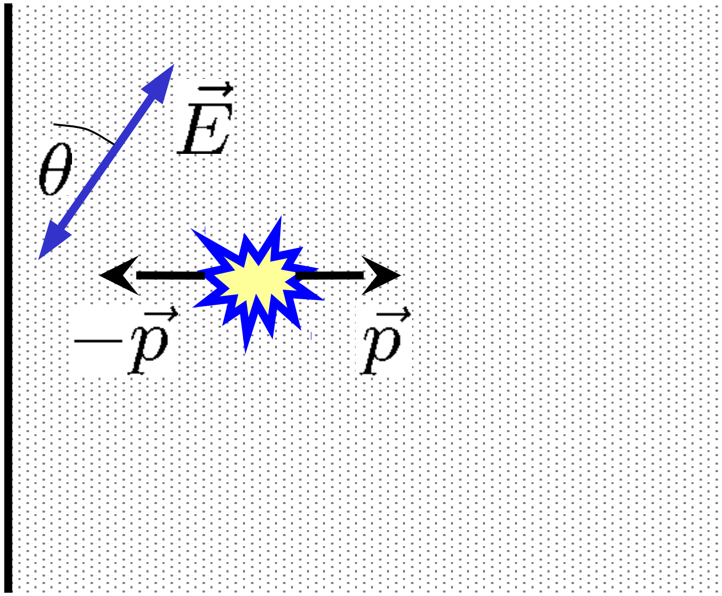
e and *h* will never meet



rough edge, oblique incidence:

backscattering is possible

Dependence on the polarization



Matrix element
for excitation of the *e*-*h* pair:

$$\mathcal{M} \propto [\vec{E} \times \vec{p}]_z \propto \cos\theta$$

D peak intensity
(unpolarized detector): $I_D \propto \cos^2\theta$

What happens when $\theta = \pi/2$ ($\vec{E} \perp$ edge)?

In all experiments a finite value is seen

Dependence on the polarization

The most general form for unpolarized detection:

$$I_D(\theta) = I_{\max} \cos^2(\theta - \theta_{\max}) + I_{\min} \cos^2(\theta - \theta_{\min})$$

1. Perfect armchair edge: $\theta_{\max} = 0,\quad I_{\min}/I_{\max} \sim \omega_{\mathrm{ph}}^2/\omega_{\mathrm{in}}^2$

symmetry

quantum uncertainty

2. Atomically rough edge: $\theta_{\max} = 0,\quad I_{\min}/I_{\max} \sim 1$

symmetry

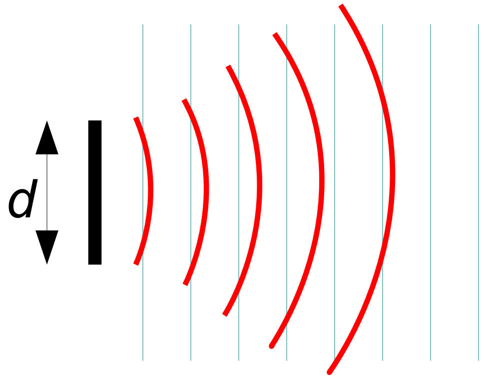
diffuse scattering

Basko, PRB **79**, 205428 (2009)

Dependence on the polarization

3. A single armchair segment $d \gg v/\omega_{in}$

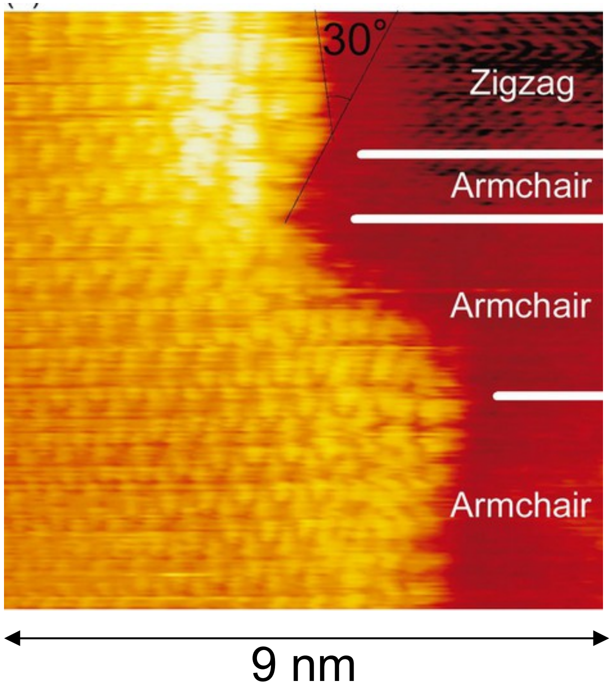
STM: Kobayashi *et al.*,
PRB **71**, 193406 (2005)



Diffraction from the Huygens-Fresnel principle for the Dirac equation:

$$\psi(\vec{r}) = \int_{\text{boundary}} \vec{n} \cdot [iv\, G(\vec{r} - \vec{r}\,', \epsilon)\, \vec{\sigma}\, \psi(\vec{r}\,')]\, d\vec{r}\,'$$

$\theta_{\max}$ along the segment, $\dfrac{I_{\min}}{I_{\max}} \sim \dfrac{v/\omega_{\text{in}}}{d}$

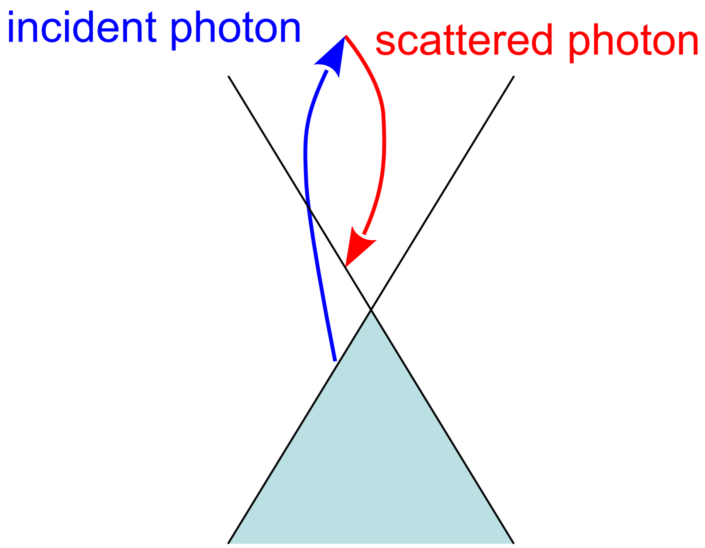


Basko, PRB **79**, 205428 (2009)

Small I_{min}/I_{max} → good quality of the edge

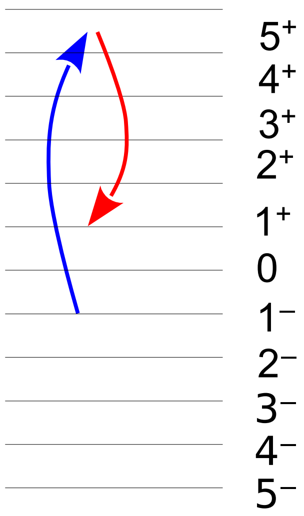
Raman scattering on electronic excitations

Without magnetic field



continuous structureless
spectrum

With magnetic field



discrete peaks
from inter-LL transitions

Selection rules in a magnetic field

co-circular polarization: $n \to n$
no angular momentum transfer

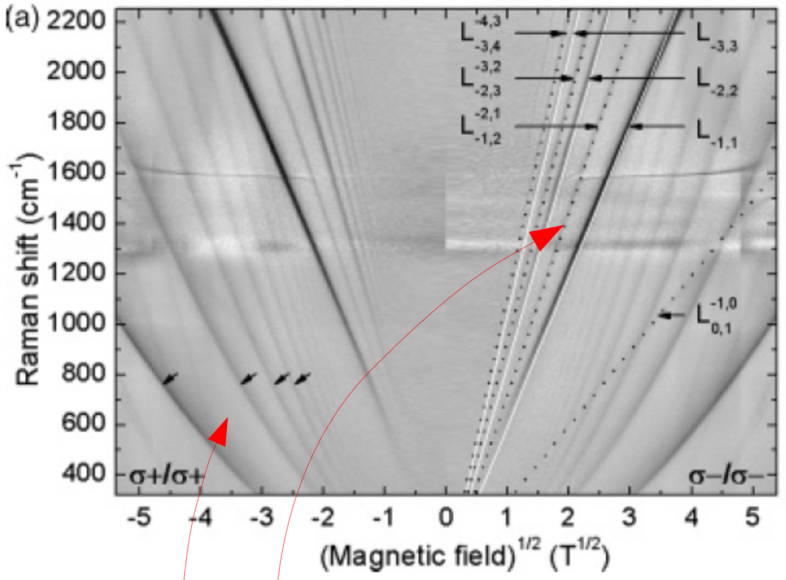
cross-circular polarization: $n \to n \pm 2$
angular momentum transfer ±2

monolayer: Kashuba and Fal'ko, PRB **80**, 241404 (2009)
bilayer: Mucha-Krucziński, Kashuba, and Fal'ko, PRB **82**, 045405 (2010)

Raman scattering on electronic excitations

magneto-Raman spectrum of graphene on graphite

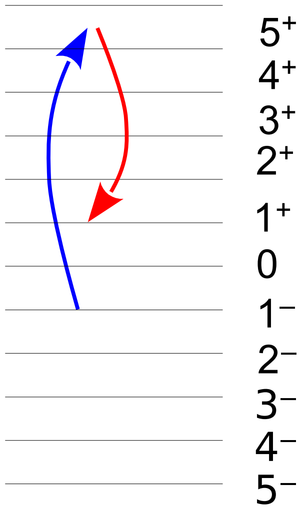
from M. Kühne *et al.*, PRB **85**, 195406 (2012)



graphene flake

bulk graphite

With magnetic field

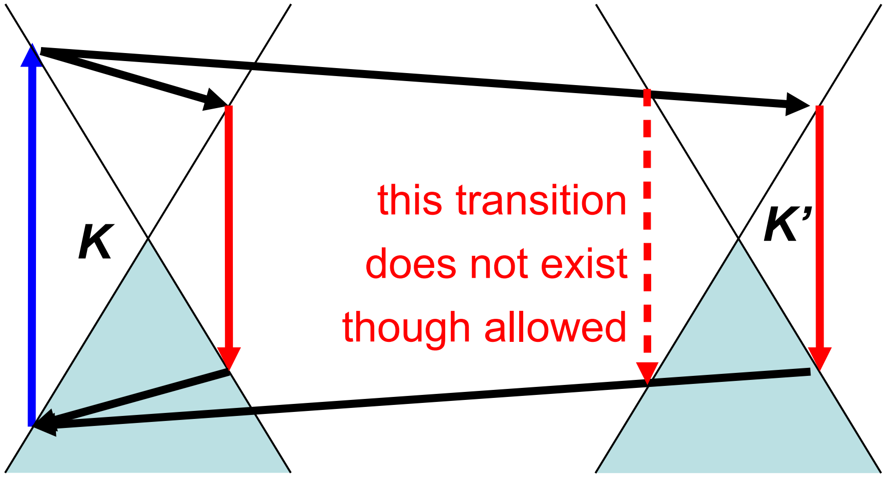


discrete peaks
from inter-LL transitions

see also Garcia-Flores *et al.*, PRB **79**, 113105 (2009)
Faugeras *et al.*, PRL **107**, 036807 (2011)
Kim *et al.*, PRB **85**, 121403(R) (2012)

Thank you for your attention!

Two-phonon Raman scattering: $\vec{q}, -\vec{q}$



this transition does not exist though allowed

Phonon momentum selected by the resonance condition:

$$\omega_{\mathrm{ph}}(\vec{q}) \approx \omega_{\mathrm{ph}}(\vec{K}) + v_{\mathrm{ph}}|\vec{q} - \vec{K}|$$

phonon group velocity, determined by the Kohn anomaly

2D peak position depends on the excitation energy