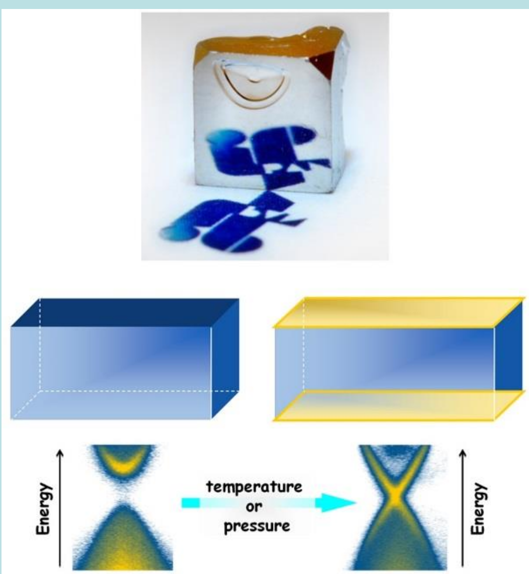
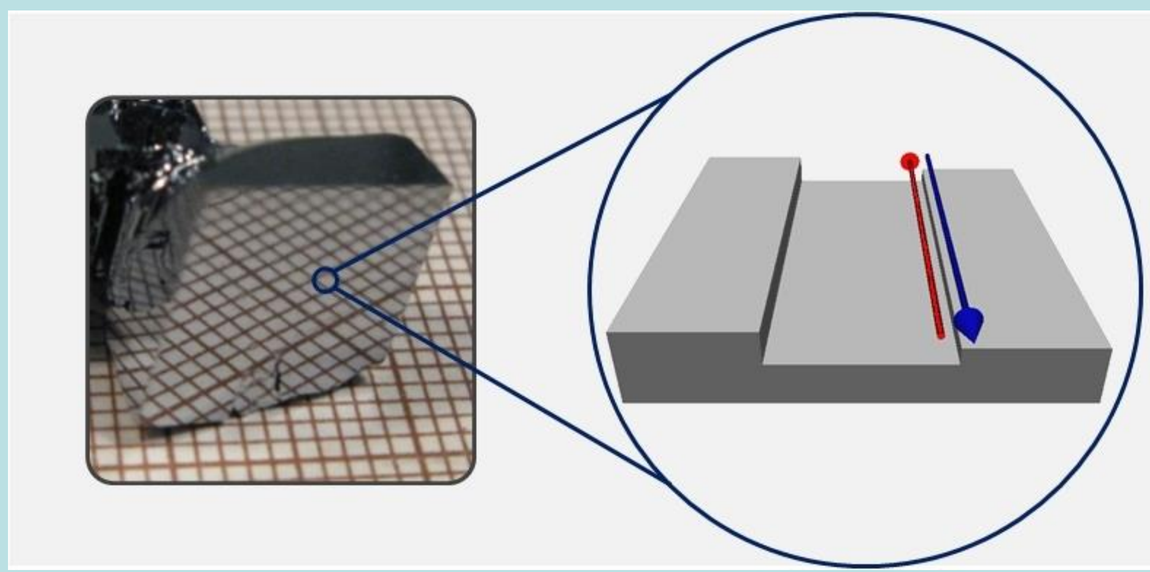
# Topological materials
# and
# topological crystalline insulators

Tomasz Story (Institute of Physics PAS, Warsaw)



# Outline

## I. Topological insulators (TI):

- TI concept and key physical factors
- Experimental techniques
- 3D and 2D materials – canonical TI
- Device ideas
- Physical classification and new topological materials

## II. Topological crystalline insulators (TCI)

- IV-VI semiconductors
- Topological protection by crystalline symmetry
- Controlling TCI state (lattice distortion, bandgap engineering)

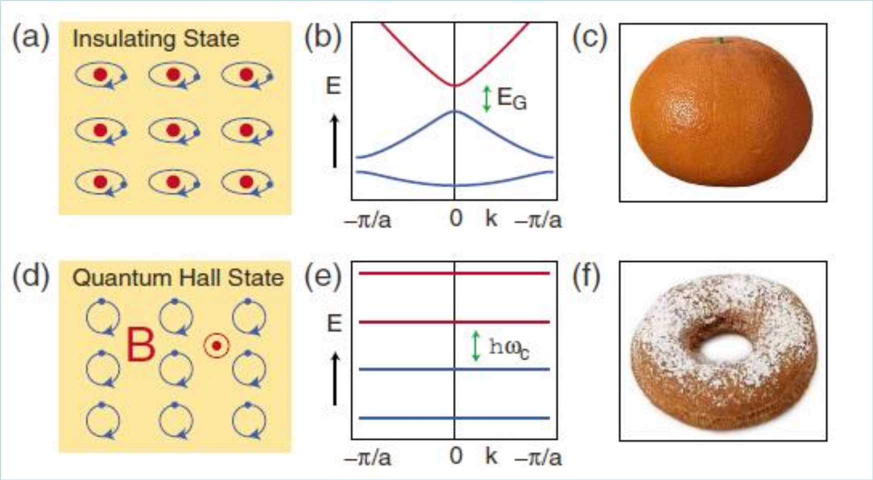
## III. Atomic steps as new 1D topological systems

## Summary

# Insulators and metals
## semiconductors - semimetals

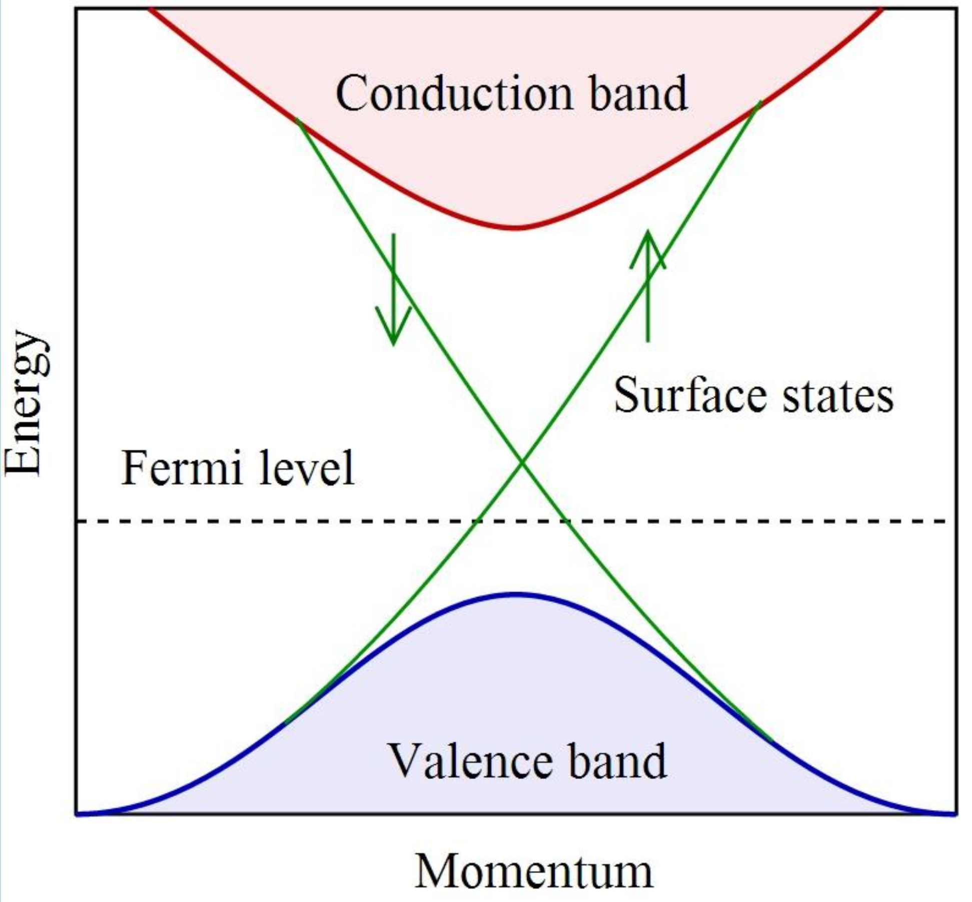
**Energy band gap $E_G$**
**Electric conductivity $\sigma_0 = \sigma(T=0)$**

**Insulators: $E_G = 0.1\text{-}10$ eV; $\sigma_0 = 0$**
**Metals: $E_G = 0$; $\sigma_0$ is finite**




M.Z. Hasan, C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010)

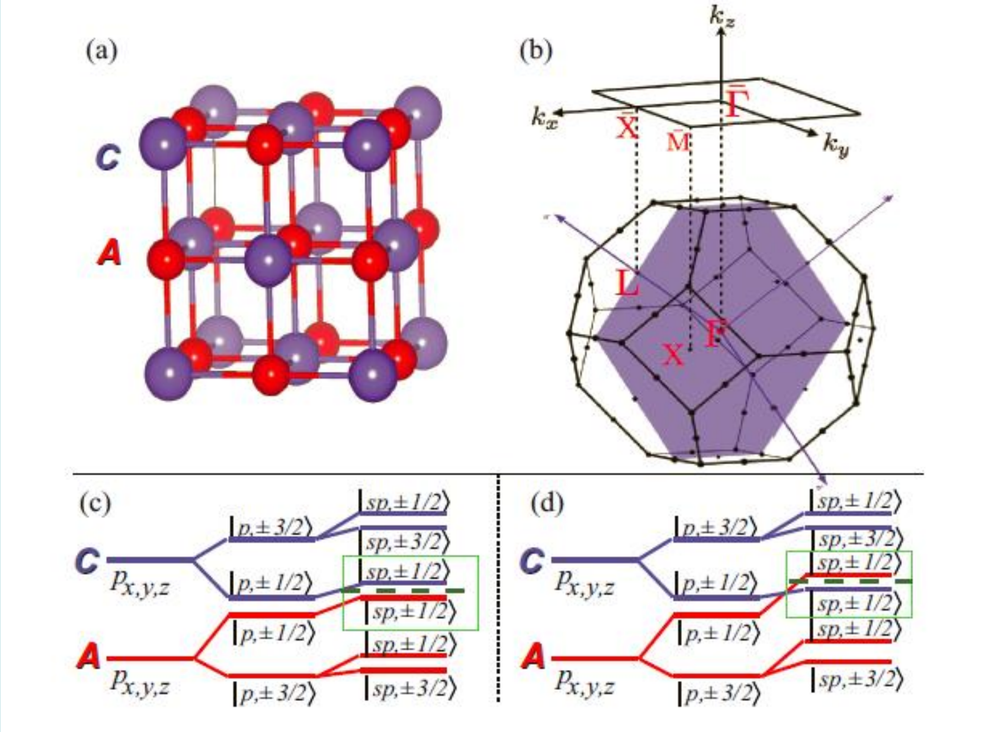
# Topological insulators: physical factors




- **Inverted c-band and v-band symmetry**
- **Strong spin-orbit coupling $E_{so} \approx E_G$**
- **Odd number of Dirac cones**
- **Metallic, helical Dirac-like electronic surface states**
- **Topological protection by time reversal symmetry**

wikipedia

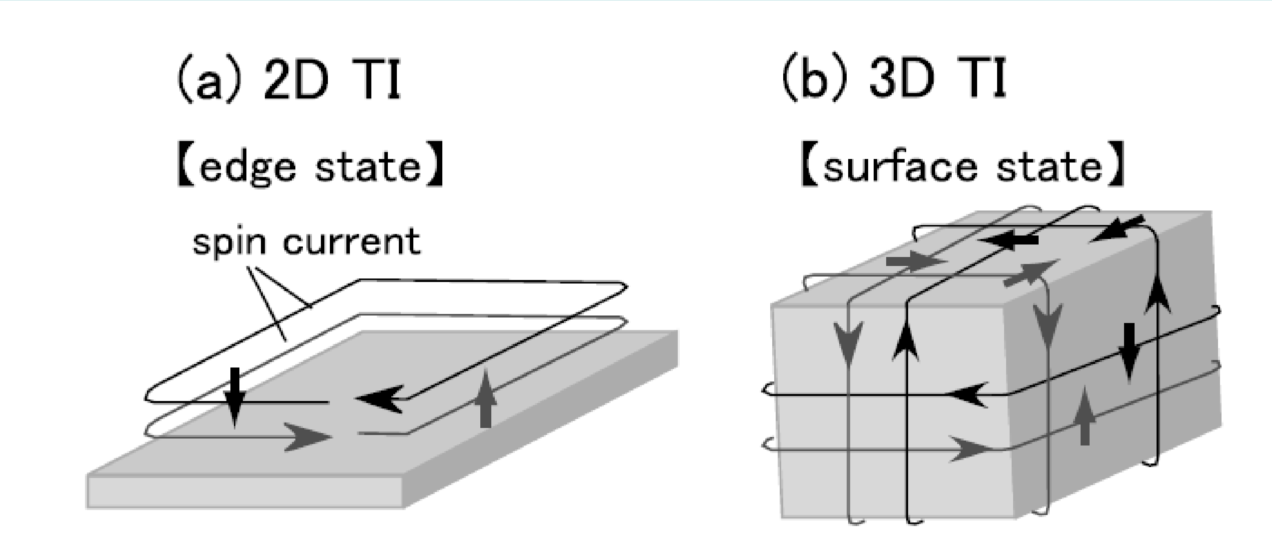
# Band inversion in semiconductors




P. Barone et al., Phys. Rev. B 88 045207 (2013)

# Topological electronic states

**Edge states in 2D heterostructures**
**Surface states in bulk crystals**




**S. Murakami, J. Phys. Conf. Ser. 302, 012019 (2011)**

# Topological electronic states experimental techniques

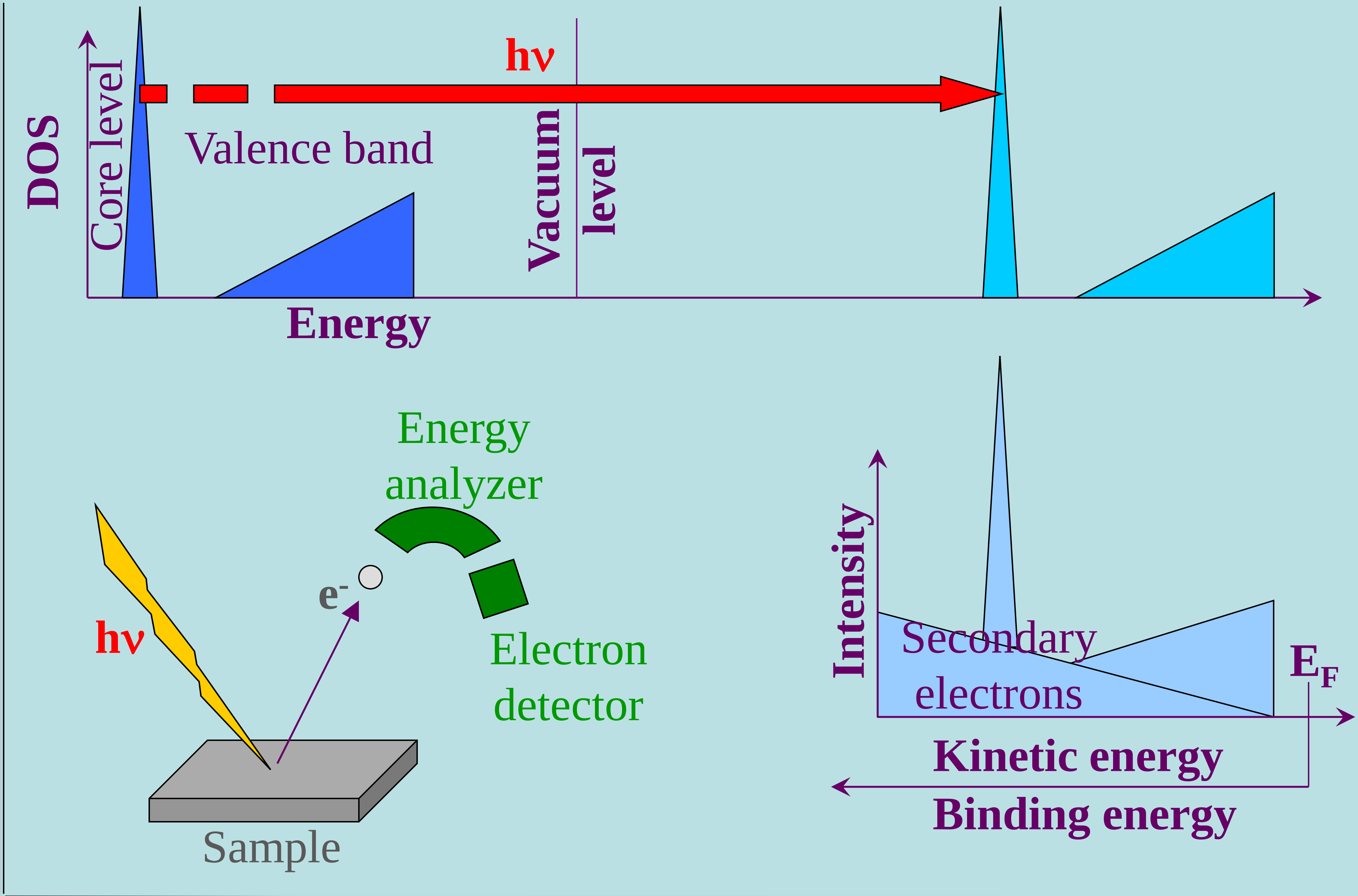
Angle- and spin-resolved photoemission spectroscopy (ARPES, SRPES)

Scanning tunneling microscopy and spectroscopy (STM/STS): conductance spectroscopy and quasiparticle interference

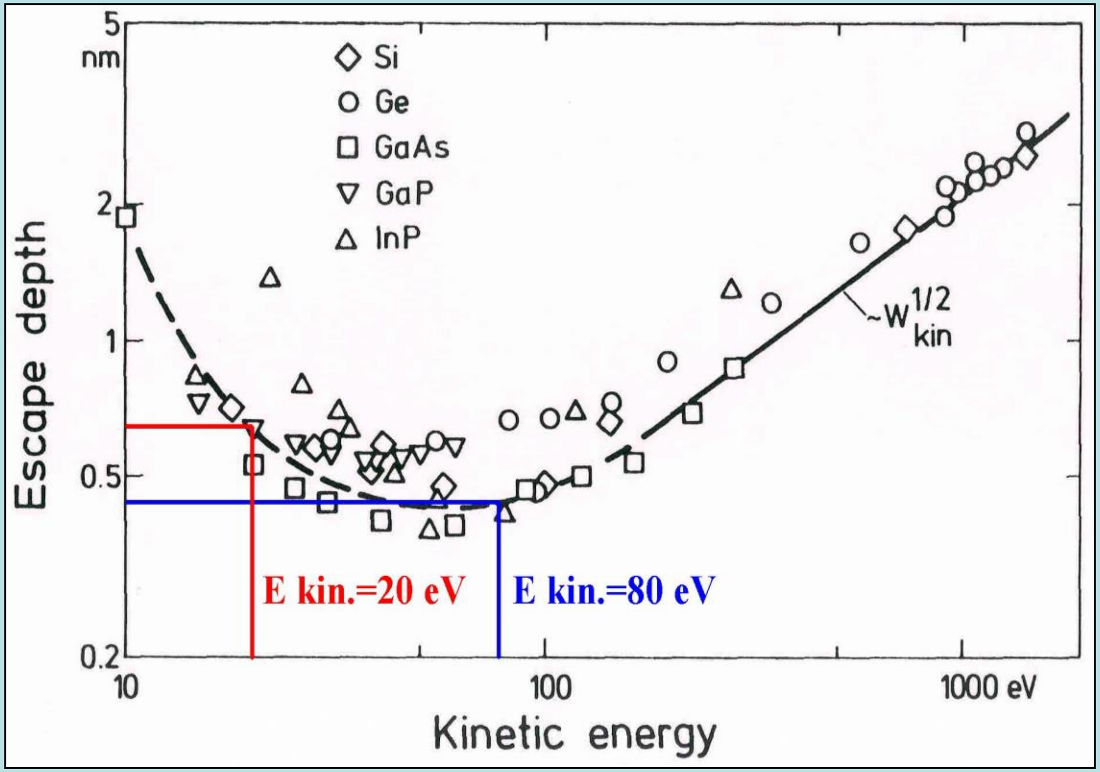
Magnetotransport: SdH oscillations, weak antilocalization

Magnetooptics

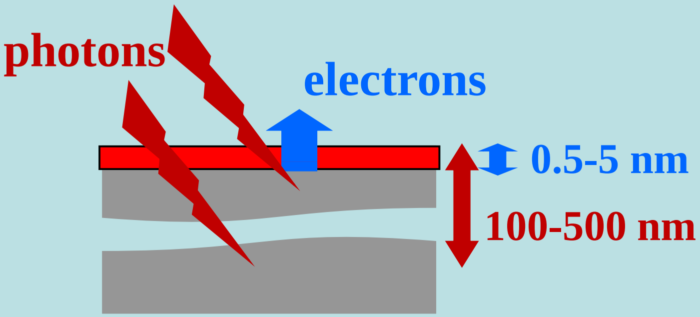
# Photoemission electron spectroscopy



# Photoemission – crystal surface sensitive technique




W. Mönch „Semiconductor surfaces and interfaces” 1993



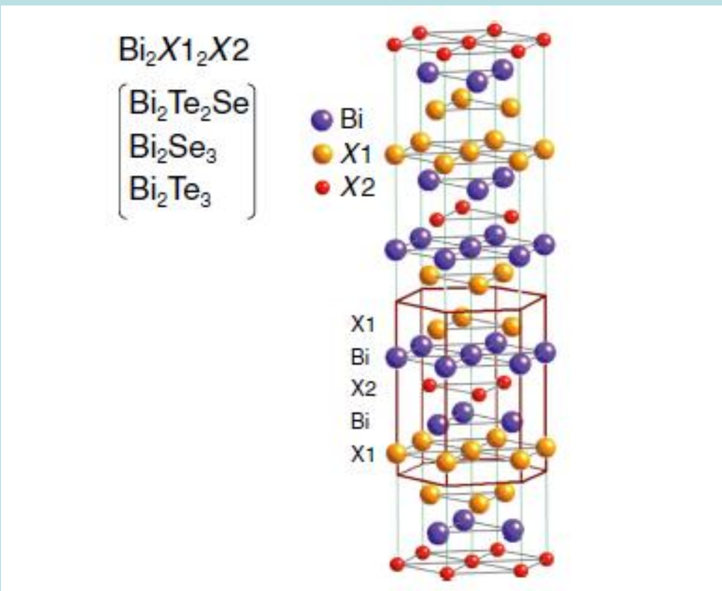
# Topological insulators: key materials

- **Bulk crystals (3D)**
- **$Bi_{1-x}Sb_x$**
- **$Bi_2Se_3$**
- **$Bi_2Te_3$**
- **(….)**

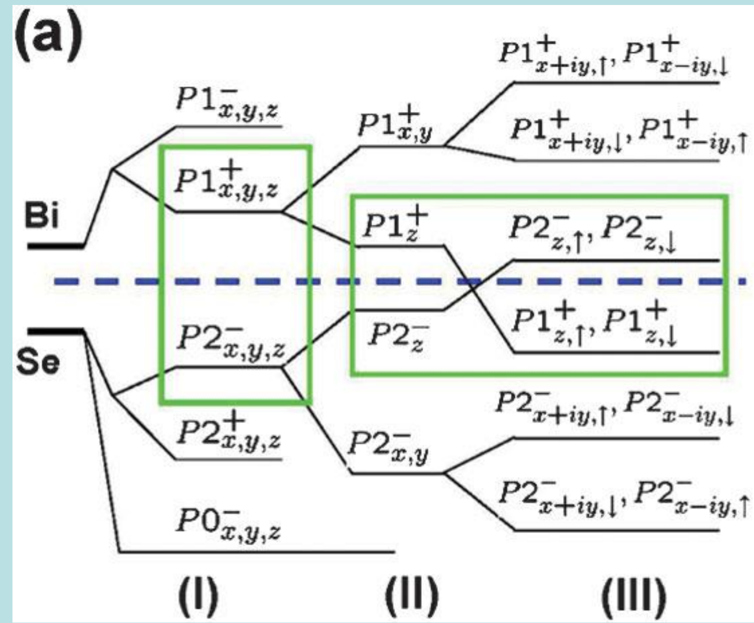
- **2D electronic systems**
- **HgTe/CdTe quantum wells**
- **InAs-GaSb heterostructures**

# 3D topological insulators

## $Bi_2Se_3$ , $Bi_2Te_3$ , $Sb_2Te_3$
## $Bi_2Te_2Se$




Crystal structure

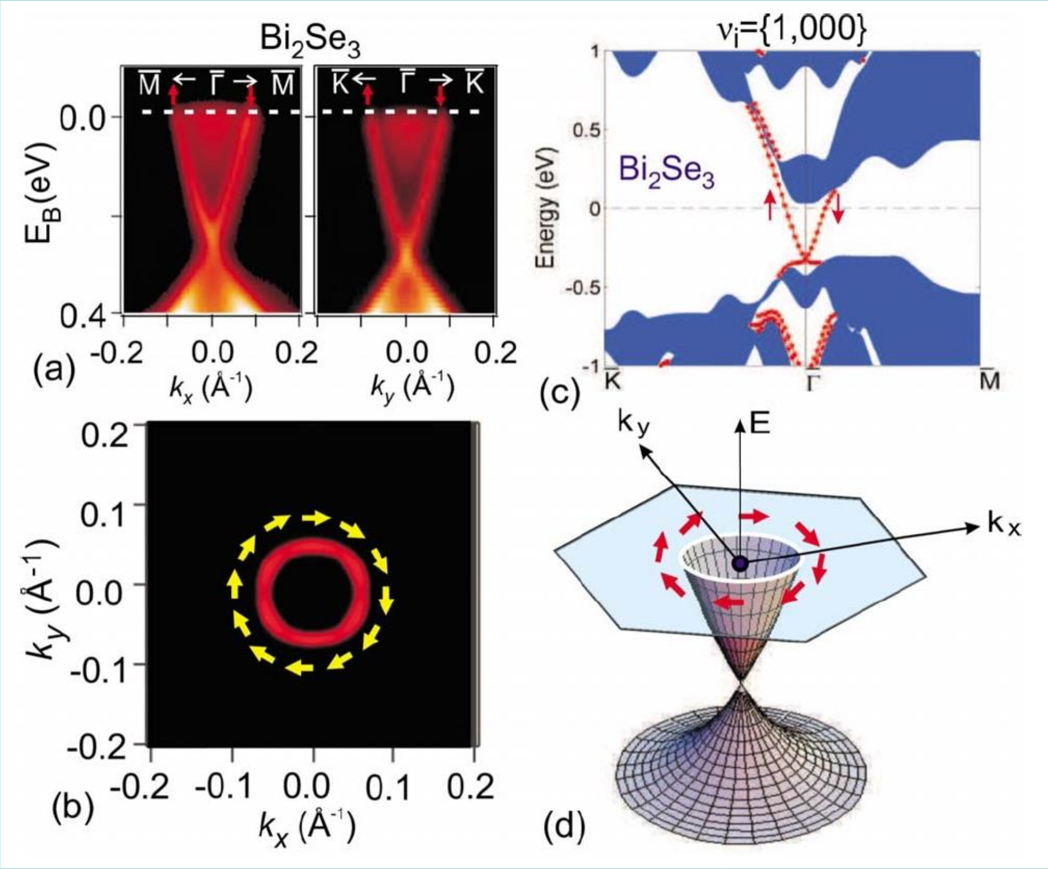



Chemical bonding | Crystal field splitting | Spin-orbit coupling

H. Zhang et al., Nat. Phys. **5**, 438 (2009)

**Electronic structure**

# Model topological insulator $Bi_2Se_3$

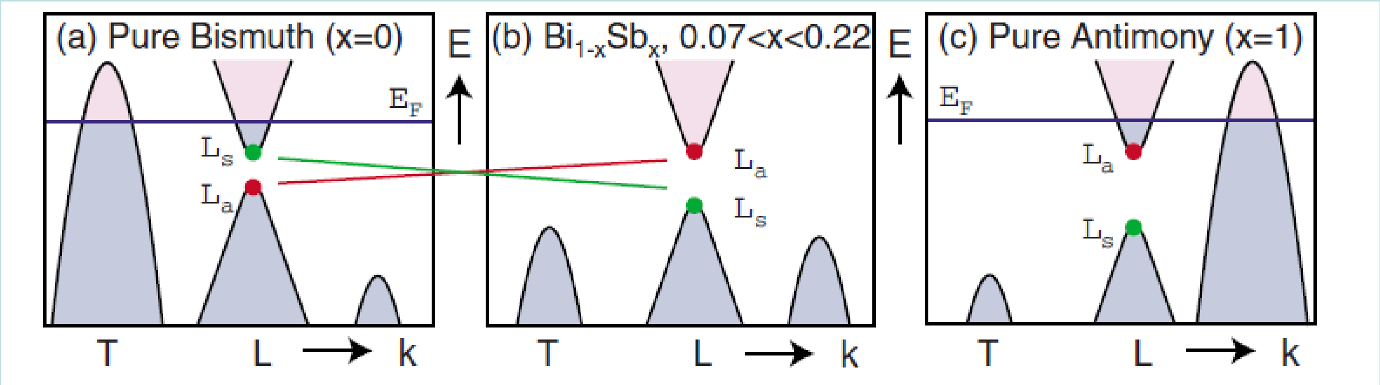



M.Z. Hasan, C.L. Kane, Rev. Mod. Phys. 82, 3045 (2010)

# 3D topological insulators

**Bi – semimetal with strong spin-orbit coupling**

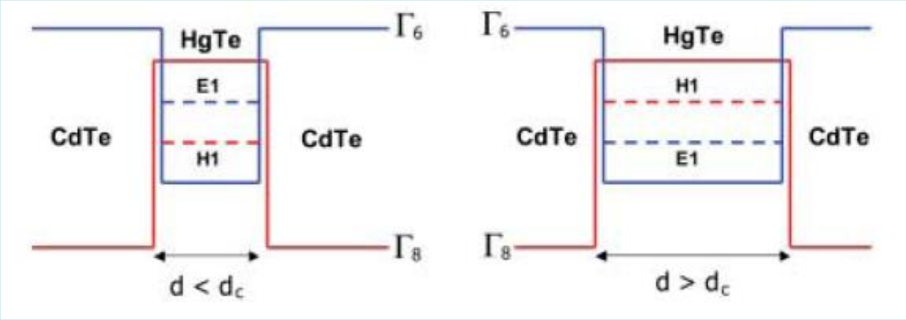
**$Bi_{1-x}Sb_x$ – semiconductor alloy – thermoelectric material**

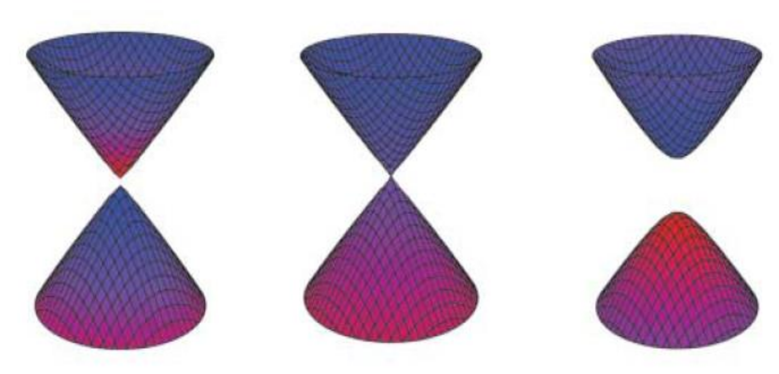


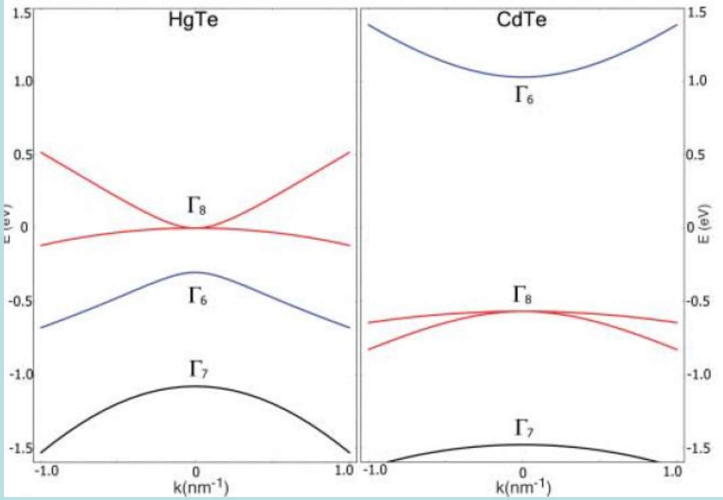

**L. Fu & C. Kane theory for $Bi_{1-x}Sb_x$ (PRB 2007)**

**Experimental verification: Hsieh et al., Nature 2008**

# HgTe/CdTe quantum wells:
# 2D topological insulators
# Quantum spin Hall (QSH) systems



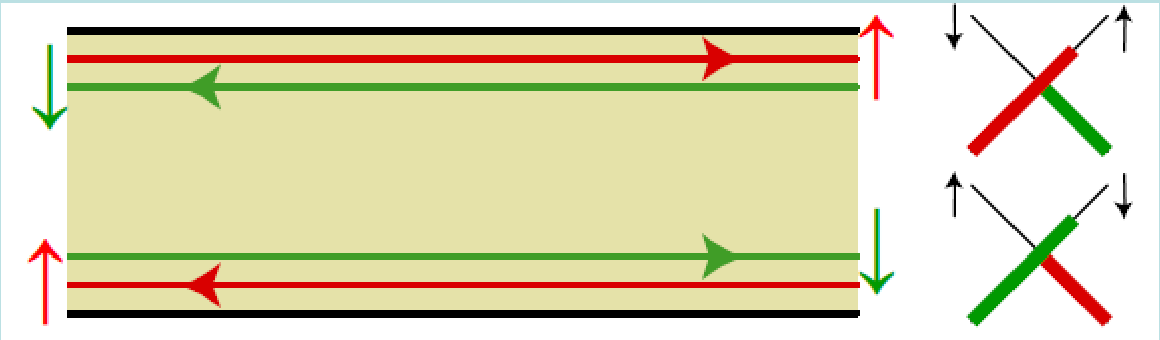



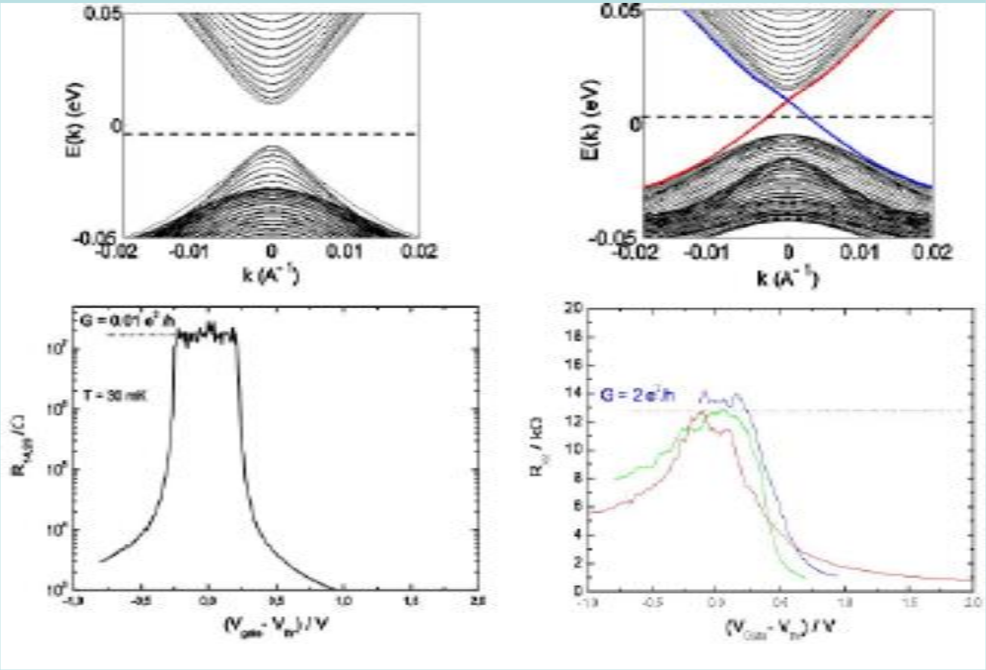



**Inversion of band edges determines the topological properties**

B. Andrei Bernevig, Taylor L. Hughes, Shou-Cheng Zhang, Science 2006

# Experimental observation of topological edge states in HgTe/CdTe (QSH) (Konig et al, Science 2007)

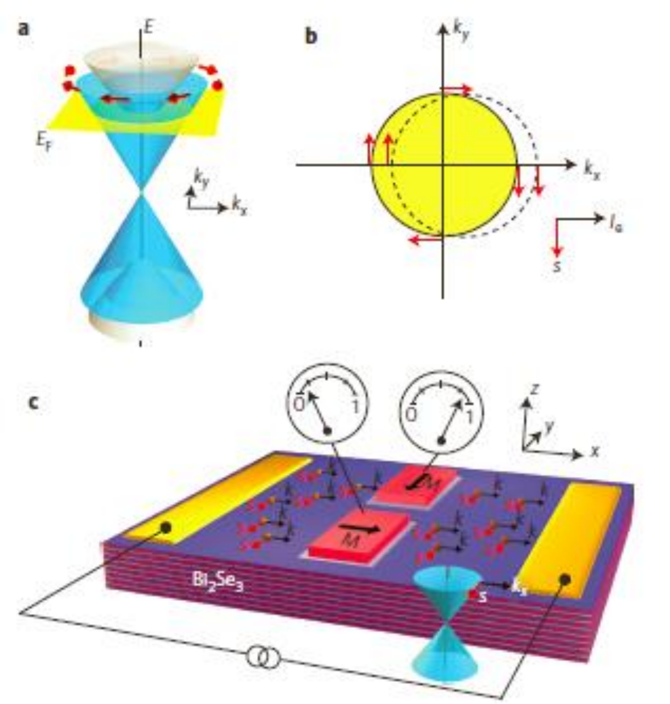





**Topological protection of edge states warranted by time reversal symmetry**

$$\langle \psi(k,\uparrow) | V | \psi(-k,\downarrow) \rangle = 0$$

# Generation of spin current
# Electrical detection of spin polarization

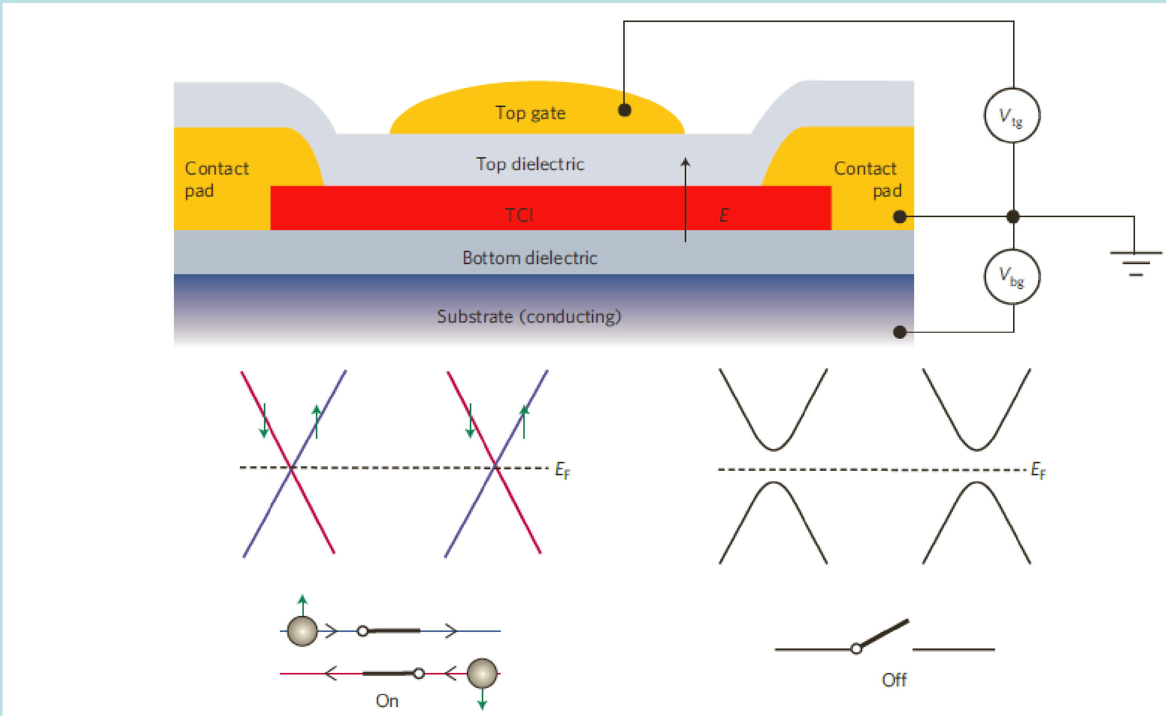



nature nanotechnology
ARTICLES
PUBLISHED ONLINE: 23 FEBRUARY 2014 | DOI: 10.1038/NNANO.2014.16


## Electrical detection of charge-current-induced spin polarization due to spin-momentum locking in $Bi_2Se_3$


C. H. Li[1]*, O. M. J. van 't Erve[1], J. T. Robinson[2], Y. Liu[3], L. Li[3] and B. T. Jonker[1]*

# Topological transistor





ARTICLES
PUBLISHED ONLINE: 22 DECEMBER 2013 | DOI: 10.1038/NMAT3828
nature materials

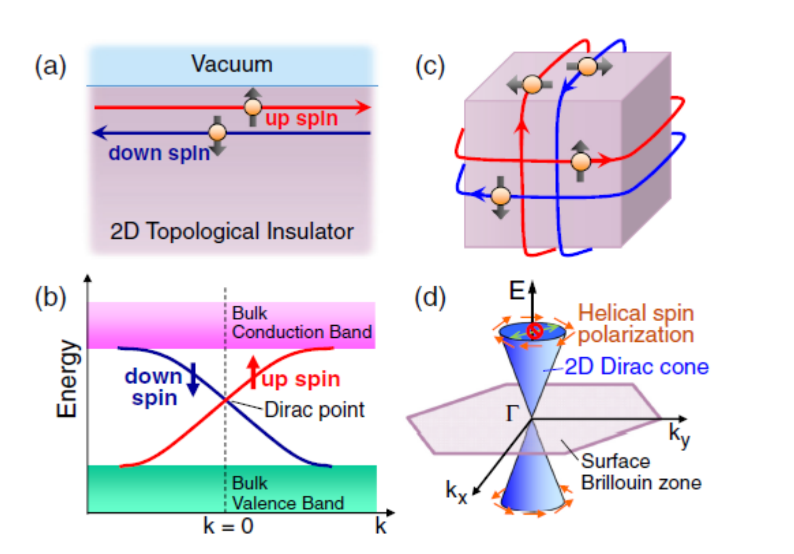

## Spin-filtered edge states with an electrically tunable gap in a two-dimensional topological crystalline insulator


Junwei Liu[1,2], Timothy H. Hsieh[2], Peng Wei[2,3], Wenhui Duan[1], Jagadeesh Moodera[2,3] and Liang Fu[2*]

# Topological materials

**Table I.** Summary of topological insulator materials that have bee experimentally addressed. The definition of (1;111) etc. is introduced in Sect. 3.7. (In this table, S.S., P.T., and SM stand for surface state, phase transition, and semimetal, respectively.)

| Type | Material | Band gap | Bulk transport | Remark | Reference |
|---|---|---|---|---|---|
| 2D, $\nu = 1$ | CdTe/HgTe/CdTe | <10 meV | insulating | high mobility | 31 |
| 2D, $\nu = 1$ | AlSb/InAs/GaSb/AlSb | ~4 meV | weakly insulating | gap is too small | 73 |
| 3D (1;111) | $Bi_{1-x}Sb_x$ | <30 meV | weakly insulating | complex S.S. | 36, 40 |
| 3D (1;111) | Sb | semimetal | metallic | complex S.S. | 39 |
| 3D (1;000) | $Bi_2Se_3$ | 0.3 eV | metallic | simple S.S. | 94 |
| 3D (1;000) | $Bi_2Te_3$ | 0.17 eV | metallic | distorted S.S. | 95, 96 |
| 3D (1;000) | $Sb_2Te_3$ | 0.3 eV | metallic | heavily p-type | 97 |
| 3D (1;000) | $Bi_2Te_2Se$ | ~0.2 eV | reasonably insulating | $\rho_{xx}$ up to 6 Ω cm | 102, 103, 105 |
| 3D (1;000) | $(Bi,Sb)_2Te_3$ | <0.2 eV | moderately insulating | mostly thin films | 193 |
| 3D (1;000) | $Bi_{2-x}Sb_xTe_{3-y}Se_y$ | <0.3 eV | reasonably insulating | Dirac-cone engineering | 107, 108, 212 |
| 3D (1;000) | $Bi_2Te_{1.6}S_{1.4}$ | 0.2 eV | metallic | n-type | 210 |
| 3D (1;000) | $Bi_{1.1}Sb_{0.9}Te_2S$ | 0.2 eV | moderately insulating | $\rho_{xx}$ up to 0.1 Ω cm | 210 |
| 3D (1;000) | $Sb_2Te_2Se$ | ? | metallic | heavily p-type | 102 |
| 3D (1;000) | $Bi_2(Te,Se)_2(Se,S)$ | 0.3 eV | semi-metallic | natural Kawazulite | 211 |
| 3D (1;000) | $TlBiSe_2$ | ~0.35 eV | metallic | simple S.S., large gap | 110–112 |
| 3D (1;000) | $TlBiTe_2$ | ~0.2 eV | metallic | distorted S.S. | 112 |
| 3D (1;000) | $TlBi(S,Se)_2$ | <0.35 eV | metallic | topological P.T. | 116, 117 |
| 3D (1;000) | $PbBi_2Te_4$ | ~0.2 eV | metallic | S.S. nearly parabolic | 121, 124 |
| 3D (1;000) | $PbSb_2Te_4$ | ? | metallic | p-type | 121 |
| 3D (1;000) | $GeBi_2Te_4$ | 0.18 eV | metallic | n-type | 102, 119, 120 |
| 3D (1;000) | $PbBi_4Te_7$ | 0.2 eV | metallic | heavily n-type | 125 |
| 3D (1;000) | $GeBi_{4-x}Sb_xTe_7$ | 0.1–0.2 eV | metallic | n (p) type at $x = 0$ (1) | 126 |
| 3D (1;000) | $(PbSe)_5(Bi_2Se_3)_6$ | 0.5 eV | metallic | natural heterostructure | 130 |
| 3D (1;000) | $(Bi_2)(Bi_2Se_{2.6}S_{0.4})$ | semimetal | metallic | $(Bi_2)_n(Bi_2Se_3)_m$ series | 127 |
| 3D (1;000) | $(Bi_2)(Bi_2Te_3)_2$ | ? | ? | no data published yet | 128 |
| 3D TCI | SnTe | 0.3 eV (4.2 K) | metallic | Mirror TCI, $n_M = -2$ | 62 |
| 3D TCI | $Pb_{1-x}Sn_xTe$ | <0.3 eV | metallic | Mirror TCI, $n_M = -2$ | 164 |
| 3D TCI | $Pb_{0.77}Sn_{0.23}Se$ | invert with $T$ | metallic | Mirror TCI, $n_M = -2$ | 162 |
| 2D, $\nu = 1$? | Bi bilayer | ~0.1 eV | ? | not stable by itself | 82, 83 |
| 3D (1;000)? | $Ag_2Te$ | ? | metallic | famous for linear MR | 134, 135 |
| 3D (1;111)? | $SmB_6$ | 20 meV | insulating | possible Kondo TI | 140–143 |
| 3D (0;001)? | $Bi_{14}Rh_3I_9$ | 0.27 eV | metallic | possible weak 3D TI | 145 |
| 3D (1;000)? | $R$BiPt ($R$ = Lu, Dy, Gd) | zero gap | metallic | evidence negative | 152 |
| Weyl SM? | $Nd_2(Ir_{1-x}Rh_x)_2O_7$ | zero gap | metallic | too preliminary | 158 |





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http://dx.doi.org/10.7566/JPSJ.82.102001


## Topological Insulator Materials


Yoichi ANDO*

# Topological materials

**ORIGIN OF BULK BANDGAP:**

- **Semiconductor-like**
- **Electron correlations**
- **Supercoducting**
- **Magnetic field/magnetization**

**PHYSICAL SYMMETRY:**

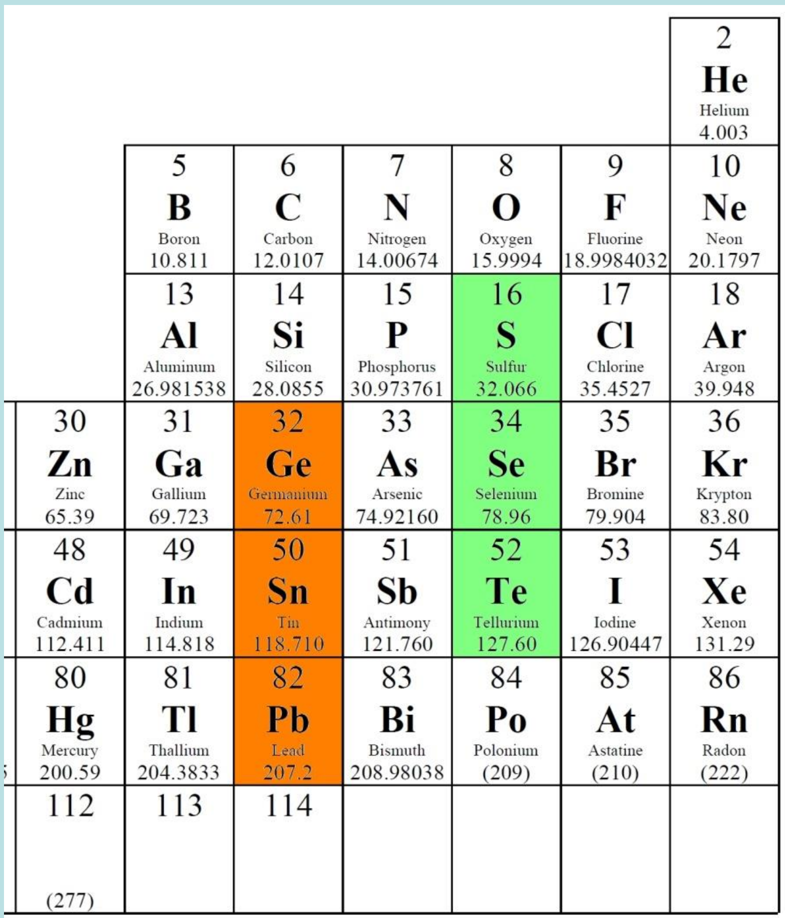
- **Time reversal symmetry (TRS)**
- **Crystal symmetry**
- **Particle-antiparticle symmetry**

- **Topological insulators ($Z_2$ class – canonical TI)**
- **Topological crystalline insulators**
- **Topological Kondo insulators**
- **Topological superconductors**
- **Quantum Hall effect and Quantum anomalous Hall effect**

- **and**
- **Topological semimetals (Dirac, Weyl)**
- **and**
- **Topological photonic, vibronic, atomic and mechanical systems**

# II. Topological crystalline insulators - TCI

## IV-VI semiconductor family

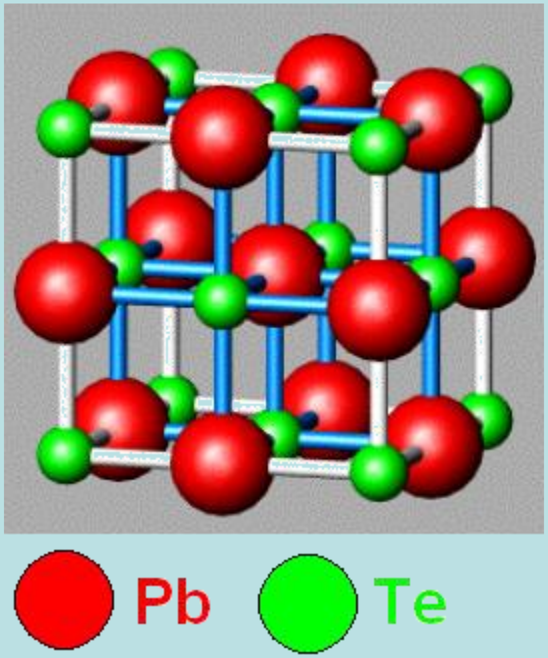


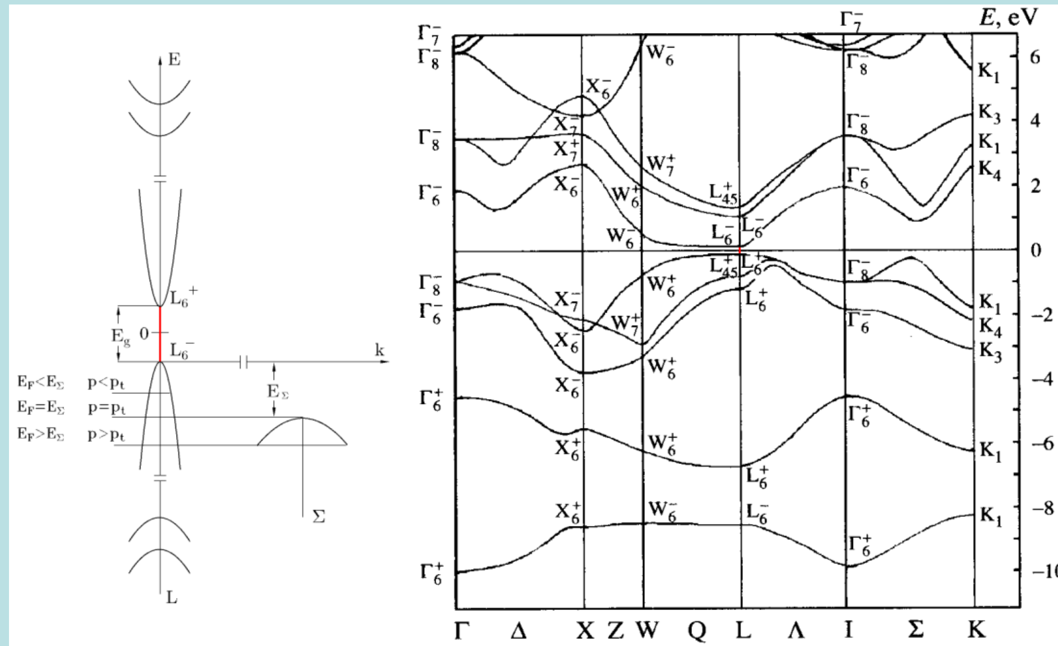

- **Binary compounds:**
- **PbTe, PbSe, PbS, SnTe, GeTe**

- **Substitutional solid solutions:**
- **$Pb_{1-x}Sn_xTe$, $Pb_{1-x}Sn_xSe$**

- **Diluted magnetic semiconductors:**
- **$Sn_{1-x}Mn_xTe$, $Ge_{1-x}Mn_xTe$**

# IV-VI semiconductors

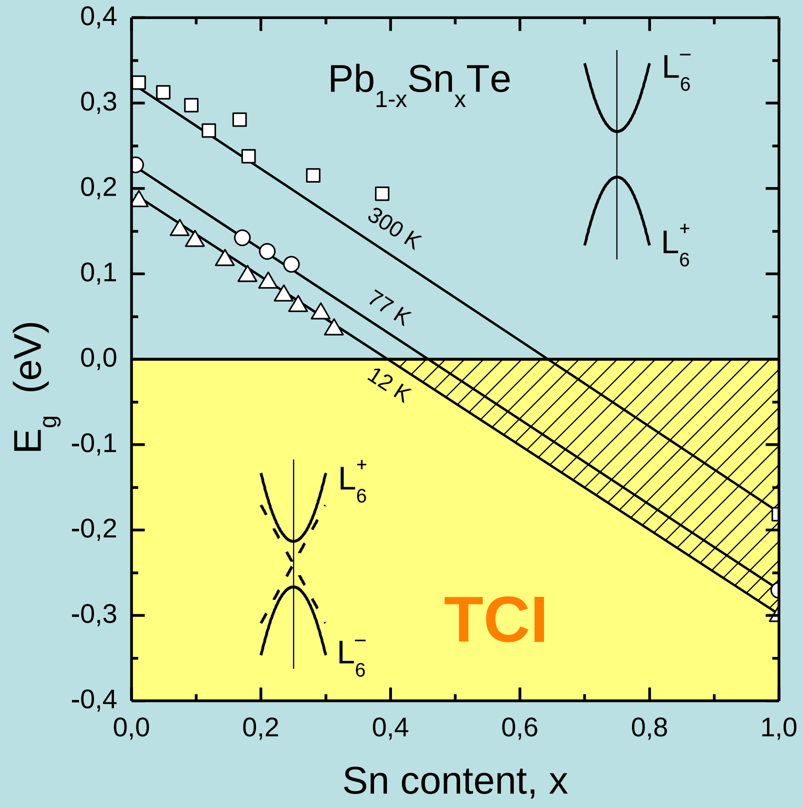


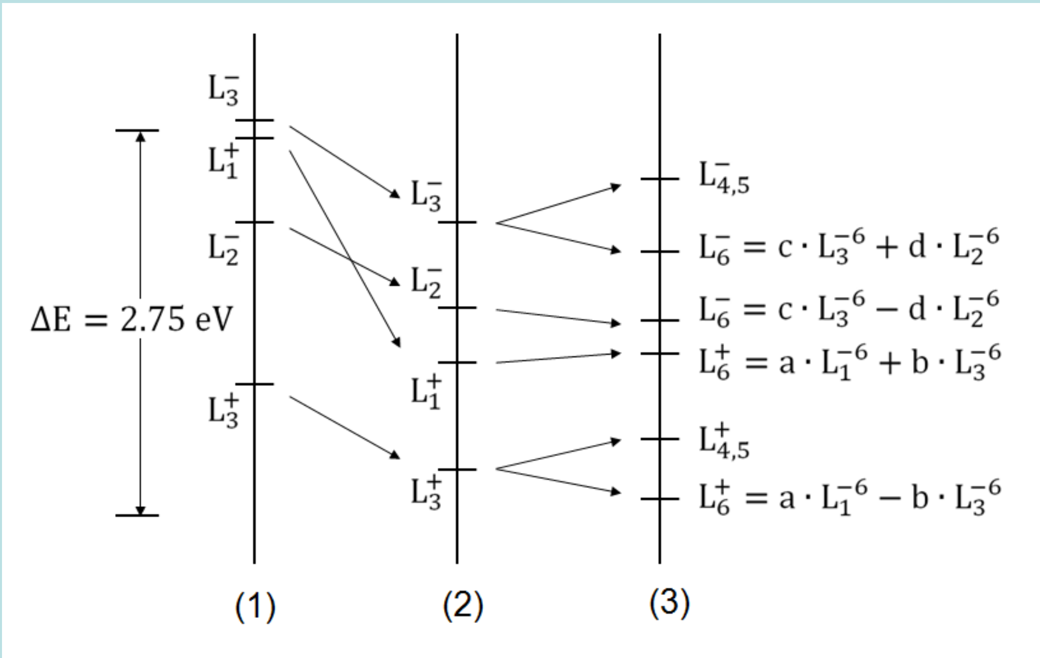

**Rock-salt crystal structure.**
**Narrow-gap materials (0-0.3 eV) with a direct gap at 4 equivalent L-points .**
**Strong (1 eV) relativistic interactions (spin-orbit and Darwin terms).**
**Small effective masses and high mobilities of electrons and holes.**
**Materials for thermoelectric generators and mid-infrared lasers and detectors.**

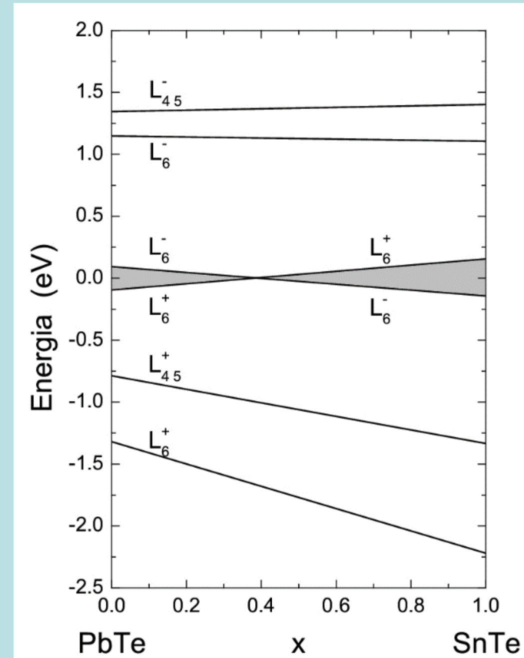
# $Pb_{1-x}Sn_xTe$ substitutional solid solutions

R. Dornhaus, G. Nimtz, and B. Schlicht,
Springer Tracts in Modern Physics vol. 98, Narrow-Gap Semiconductors
(Springer, Berlin, 1983)



# Electron band structure of IV-VI semiconductors

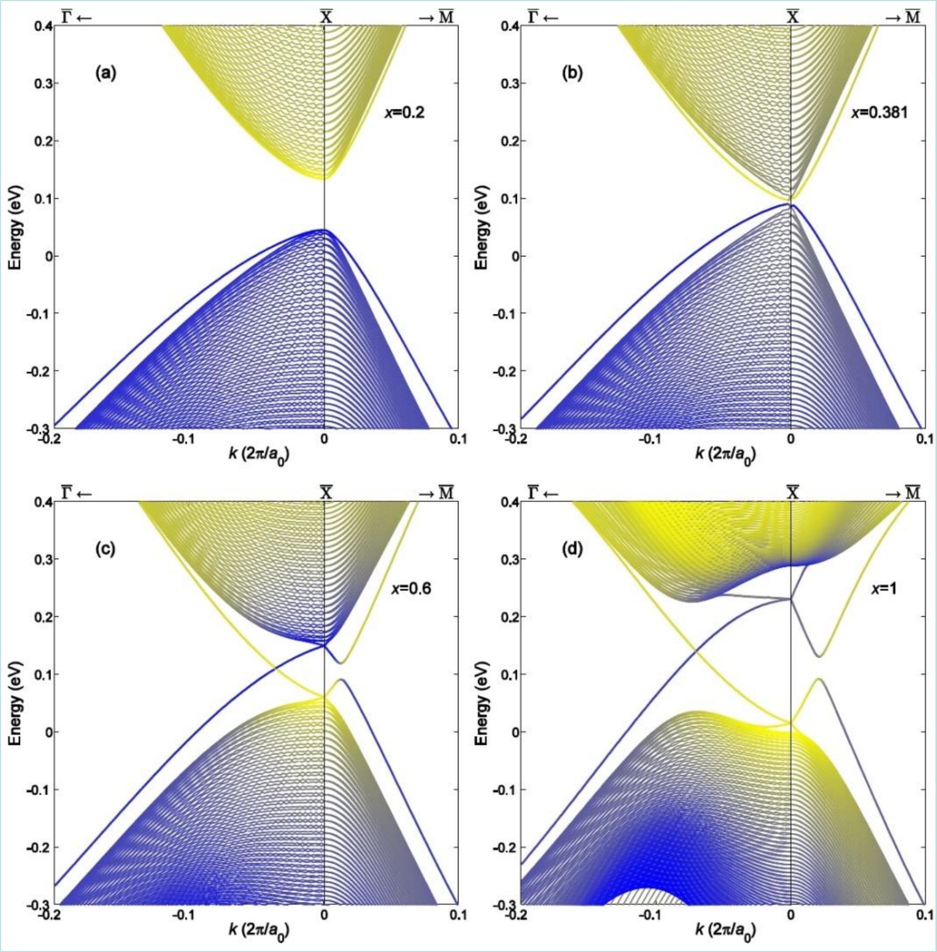



## Relativistic interactions in PbTe and $Pb_{1-x}Sn_xTe$

$$\widehat{H} = \frac{\hat{p}^2}{2m_0} + U - \frac{\hat{p}^4}{8m_0^3c^2} + \frac{\hbar^2}{8m_0^3c^2}\nabla^2 U + \frac{\hbar}{4m_0^3c^2}\hat{\sigma}(\nabla U \times \hat{p})$$

# Band structure of $Pb_{1-x}Sn_xTe$: tight binding calculations

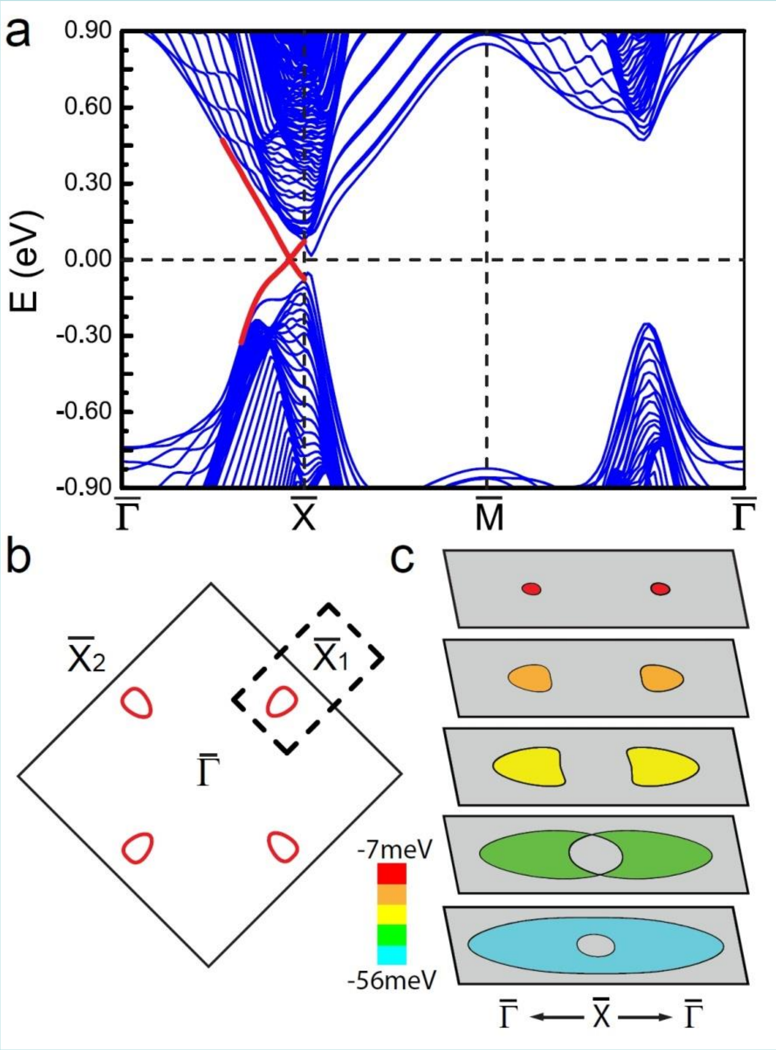



- PbSnTe in band inversion region:

- A) band insulator
- B) zero bulk band gap
- C) inverted gap - TCI
- D) SnTe – TCI

Yellow – p-type cation orbitals
Blue – p-type anion orbitals

# Topological crystalline insulators SnTe - theoretical analysis

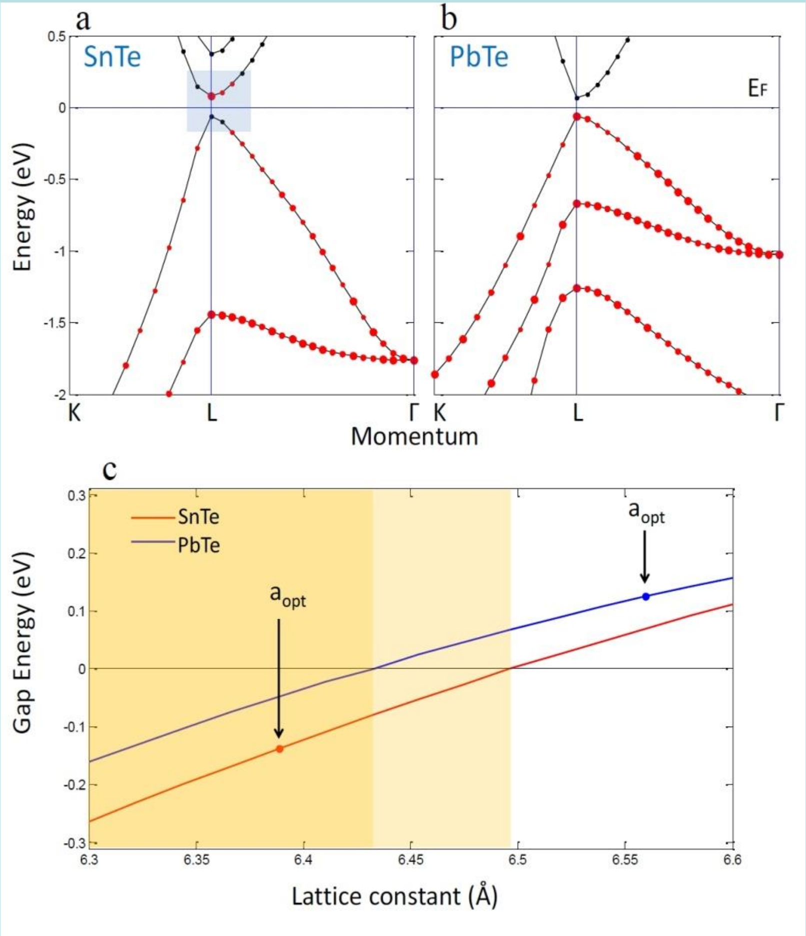



**SnTe - TCI states with 4 Dirac cones nearby X-points of the surface Brillouin zone**

**Lifshitz transition - Topological changes of Fermi surface**

**T.H. Hsieh, H. Lin, J. Liu, W. Duan, A. Bansil, L. Fu, Nature Commun. 3, 982 (2012).**

# Topological crystalline insulators: SnTe vs PbTe – theoretical analysis

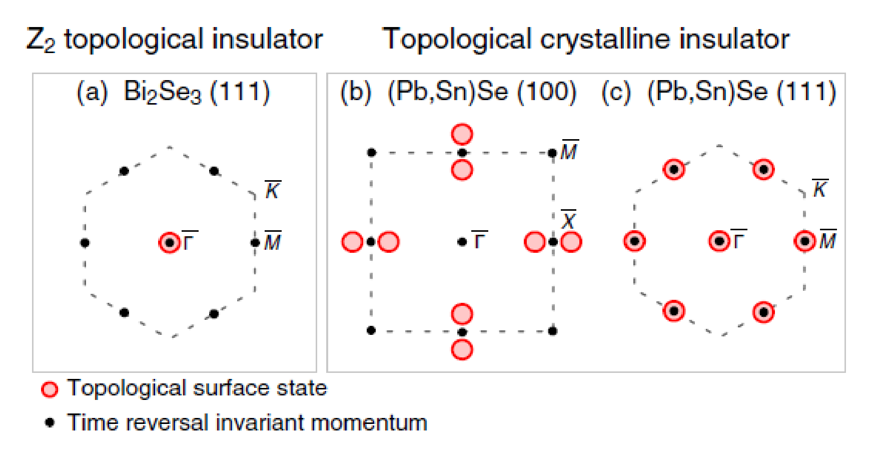



**PbTe – trivial band insulator**
**$E_G > 0$**

**SnTe – topological insulator (TCI)**
**$E_G < 0$**

**T.H. Hsieh, H. Lin, J. Liu, W. Duan, A. Bansil, L. Fu, Nature Commun. 3, 982 (2012).**

# Topological insulators (TI) vs
# Topological crystalline insulators (TCI)




**Metallic surface (or edge) states with linear energy dispersion (Dirac-like).**
**Inverted band ordering resulting from relativistic (spin-orbital) effects.**
**Topological protection.**
**Helical spin polarization.**

**Topological protection mechanism (symmetry):**
**Time reversal symmetry (in TI) - mirror-plane crystal symmetry (in TCI)**

**Dirac cones location – odd numer at TRIM points of BZ (TI) – even (in TCI)**

**Topological invariant: Chern numer, $Z_2$ (in TI) – mirror Chern numer (in TCI)**

# IV-VI topological materials

**Crystal growth:** **A. Szczerbakow**

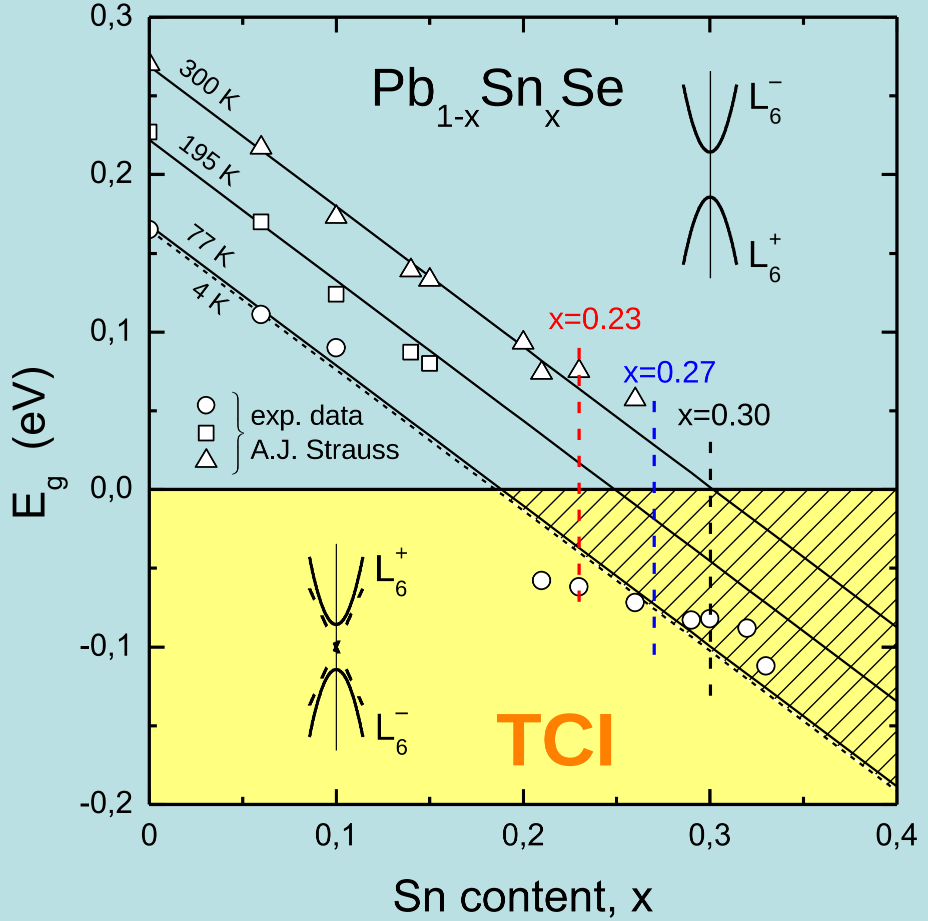
**Structural and chemical characterization:**
**J. Domagała, W. Domuchowski, E. Łusakowska, R. Minikayev, A. Reszka**

**Magneto-transport and magnetic studies – K. Dybko, W. Knoff, M. Szot**

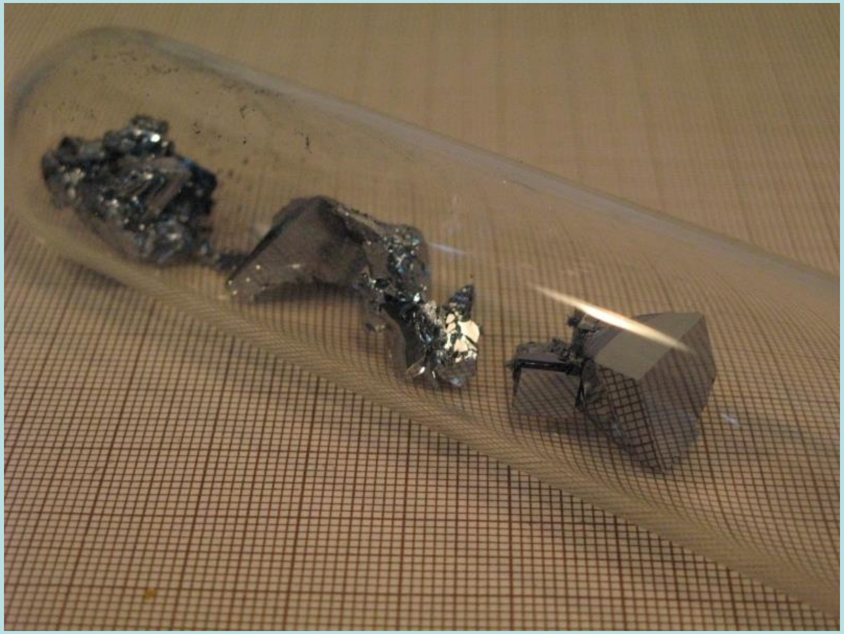
**Band structure calculations – R. Buczko, M. Galicka, P. Kacman, S. Safaei**

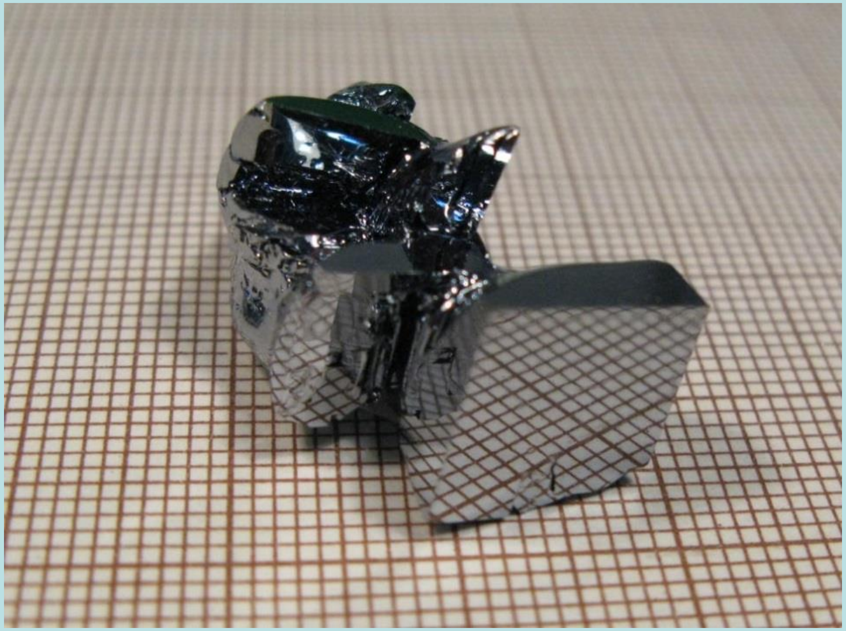
**Photoemission measurements at Lund University (synchrotron facility) and KTH Stockholm (laser facility):**
**P. Dziawa, B.J. Kowalski (IP PAS), T. Balasubramanian, C.M. Polley (Lund), M.H. Berntsen, O. Tjernberg, B.M. Wojek (KTH)**
**TS**

# $Pb_{1-x}Sn_xSe$ substitutional solid solutions



# $Pb_{1-x}Sn_xSe$ monocrystals grown by self-selecting vapor growth

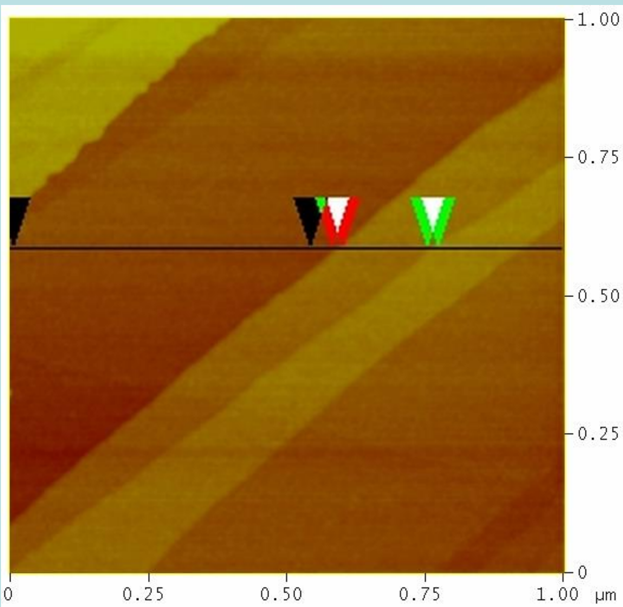


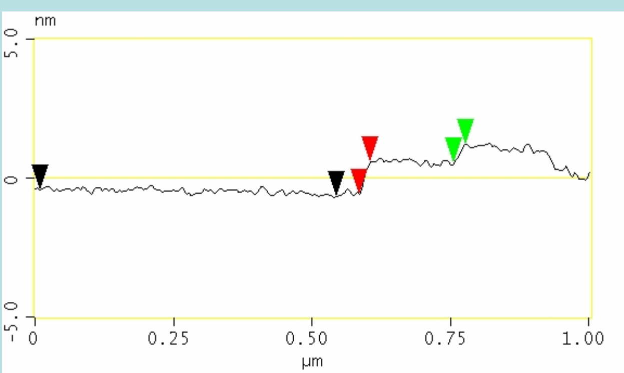


Natural (001) crystal facets – cleavage planes
Stoichiometry control of n – and p-type conductivity
Highly homogeneous chemical composition of solid solutions

A. Szczerbakow - IF PAN: J. Cryst. Growth 139, 172 (1994);

# Structural and chemical characterization

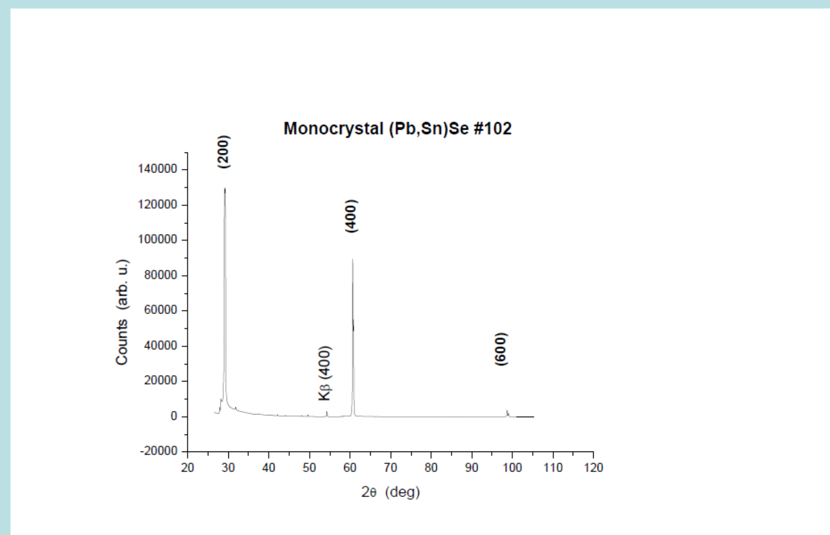


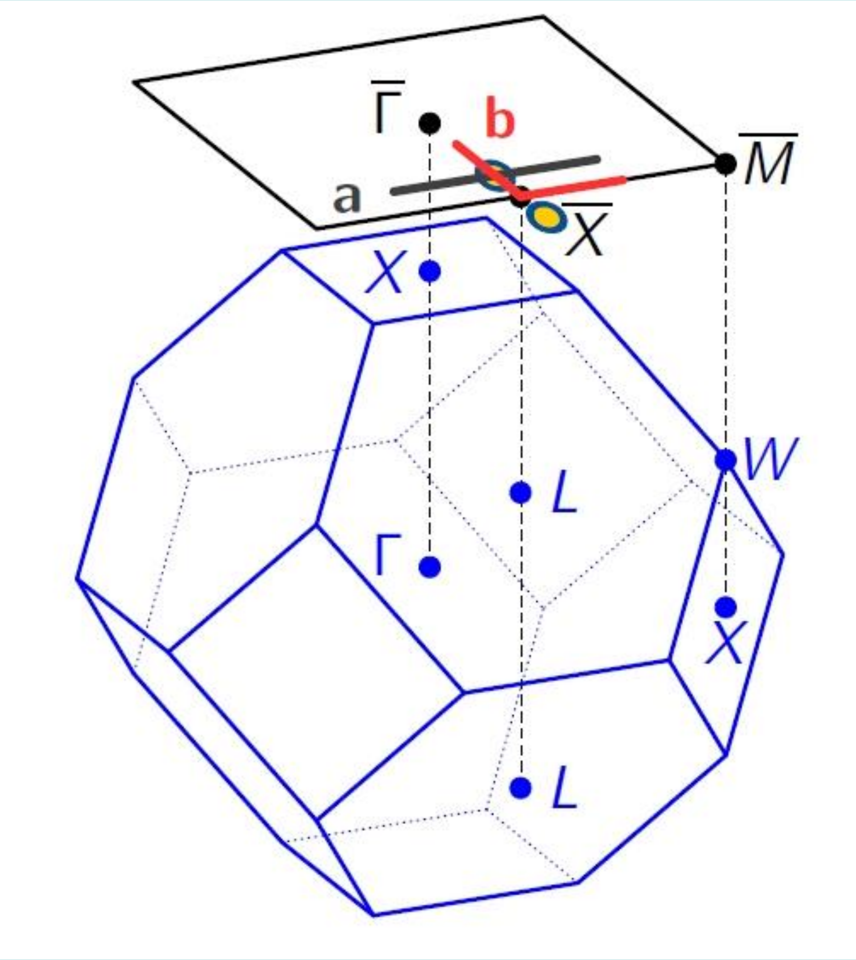

**X-ray diffraction (XRD)**

**EDX chemical analysis**

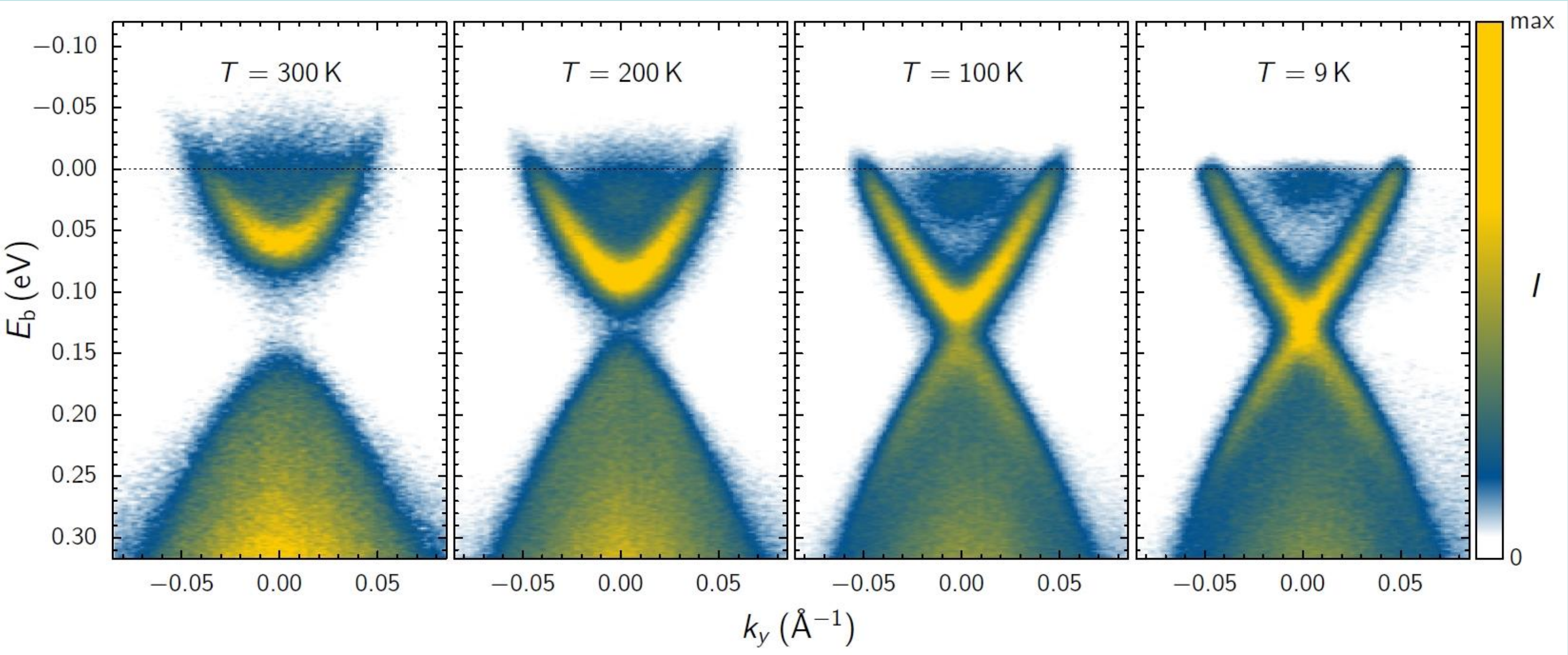
**Surface morphology analysis by AFM microscopy**



# Brillouin zone for (001) surface

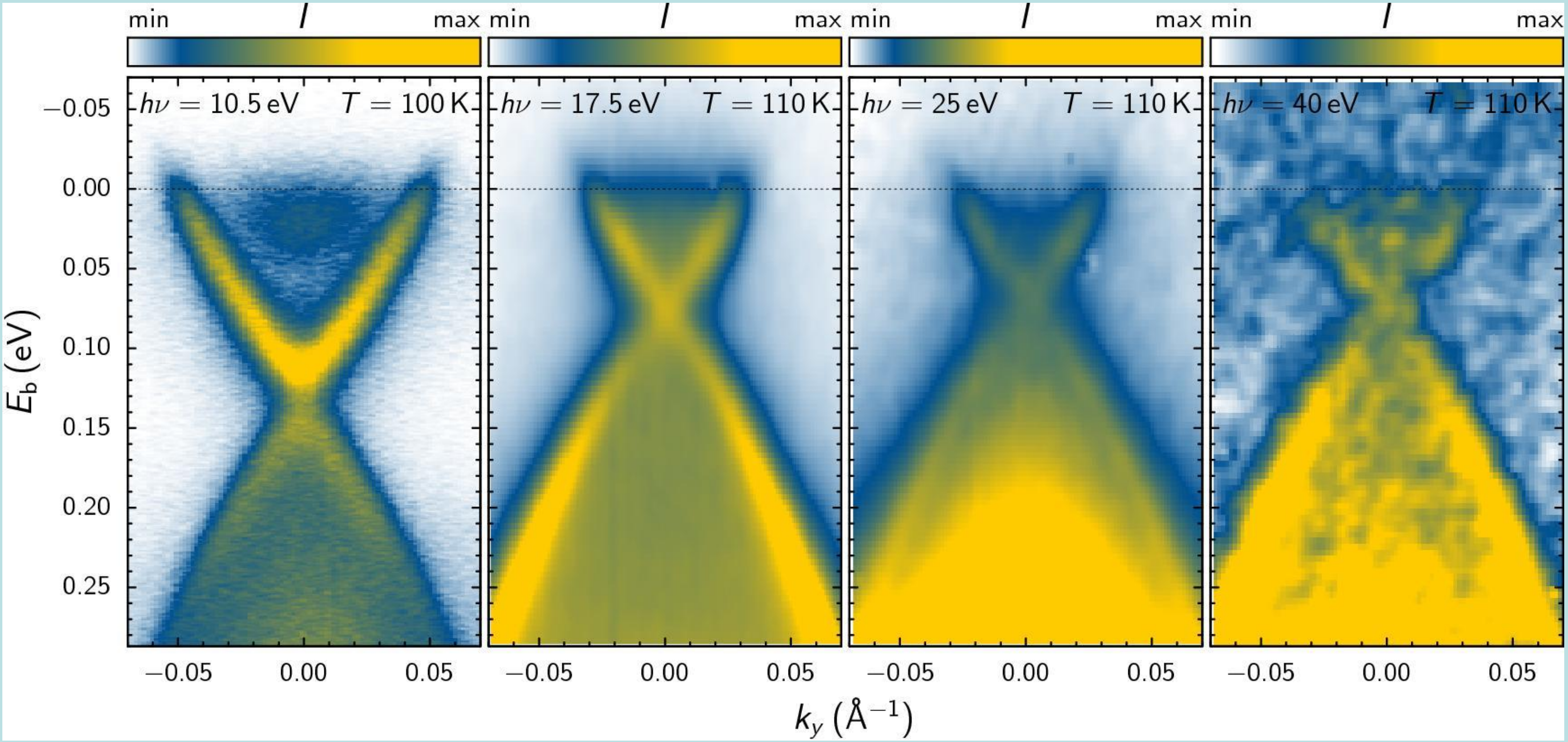


# Electron band structure of $Pb_{0.77}Sn_{0.23}Se$ ARPES experimental studies



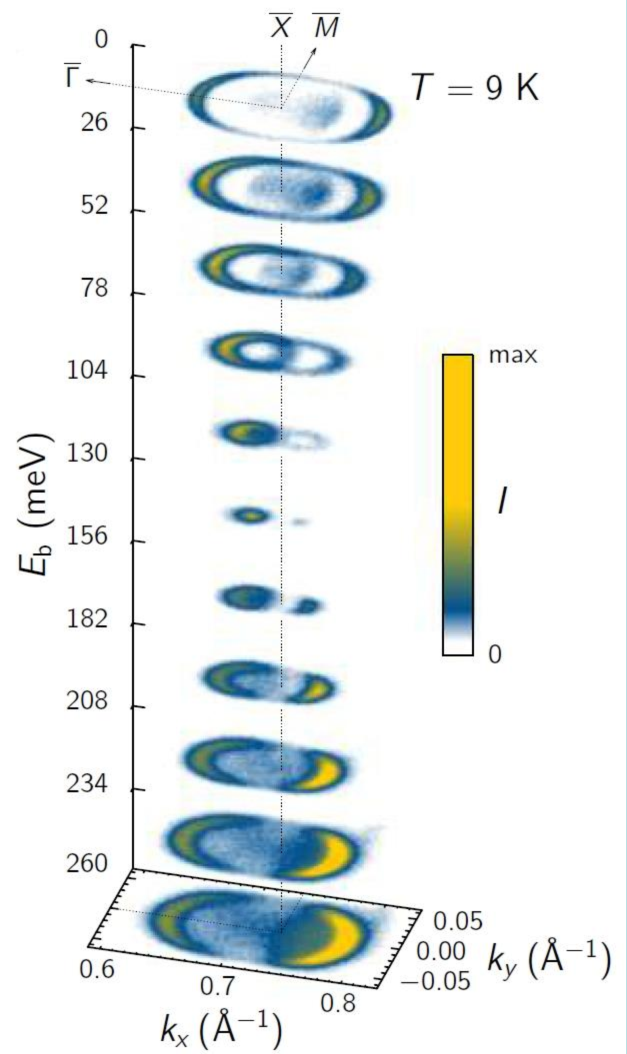

- **Energy dispersion relation for temperature varying across band inversion point**
- **P. Dziawa, B.J. Kowalski, K. Dybko, R. Buczko, A. Szczerbakow, et al. Nature Materials 11, 1023 (2012)**

# Electron band structure of $Pb_{0.77}Sn_{0.23}Se$ ARPES experimental studies



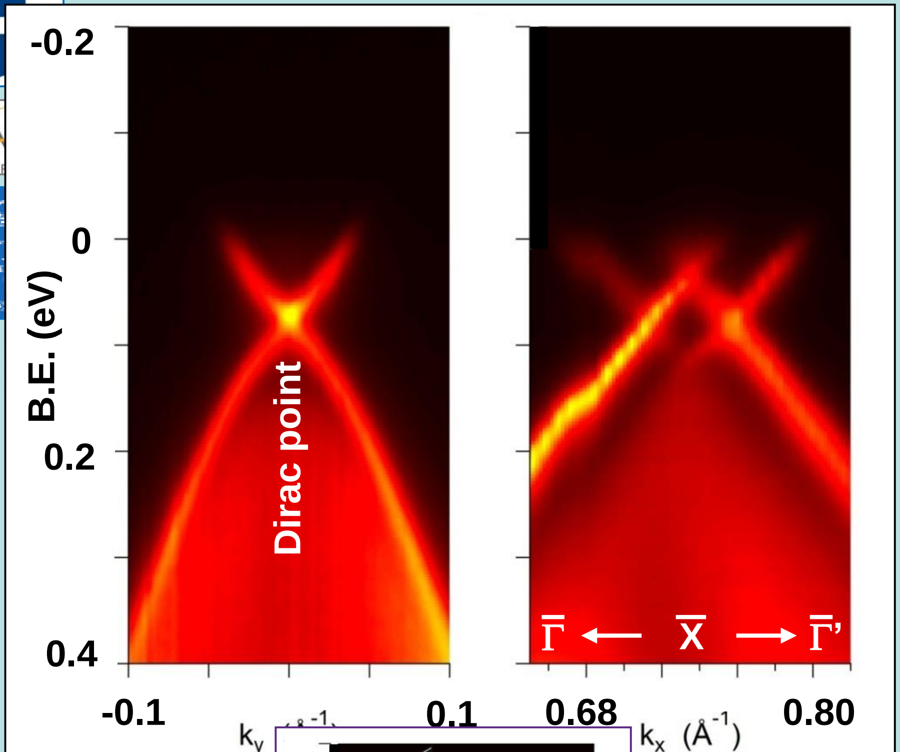

**Energy dispersion $E(k_y)$ for varying photon energy**

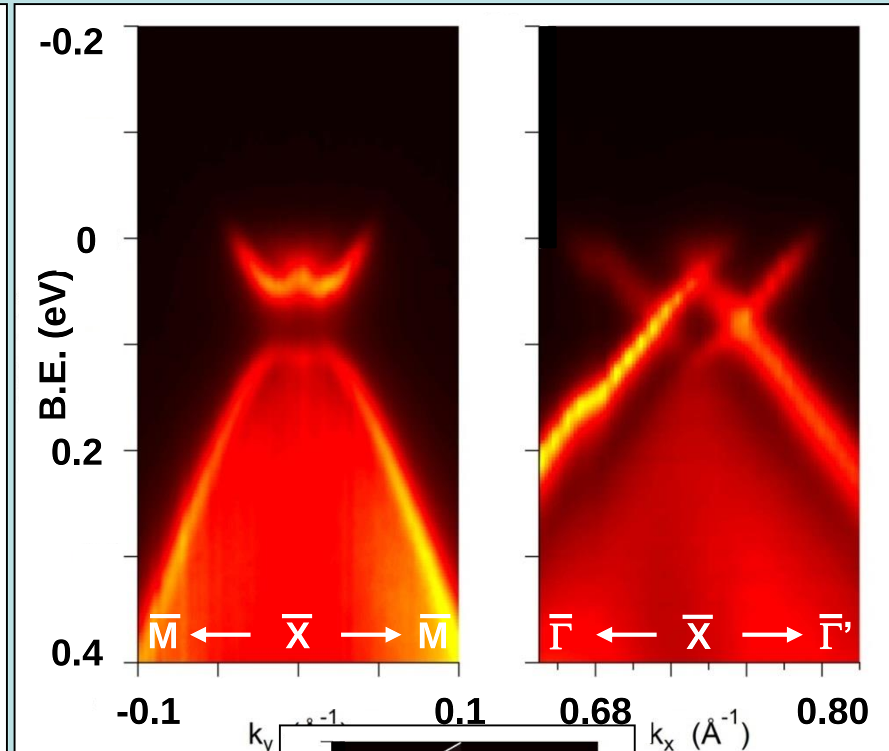
# Electronic structure - ARPES

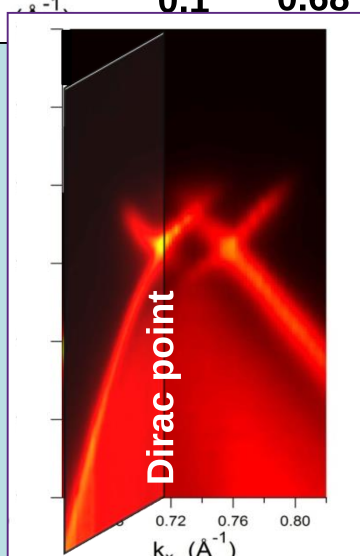


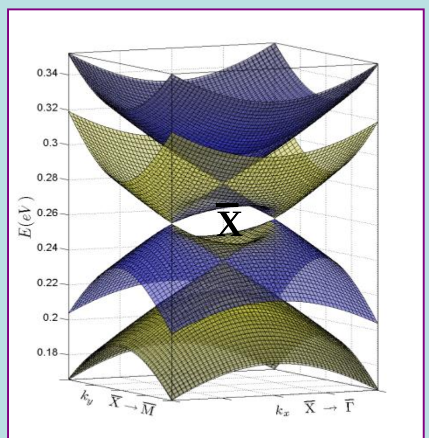

- **Fermi surface $E(k_x, k_y)$ for various binding energies $E_b$**

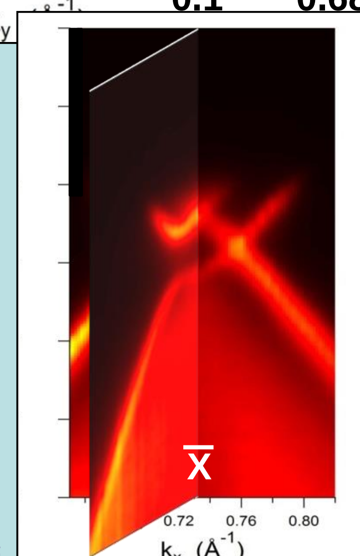
# $Pb_{0.67}Sn_{0.33}Se$, T=87 K, hν=18.5 eV




Theory-
R. Buczko, P. Kacman, S. Safaei

# Trivial insulator (PbSe) vs topological crystalline insulator ($Pb_{1-x}Sn_xSe$)

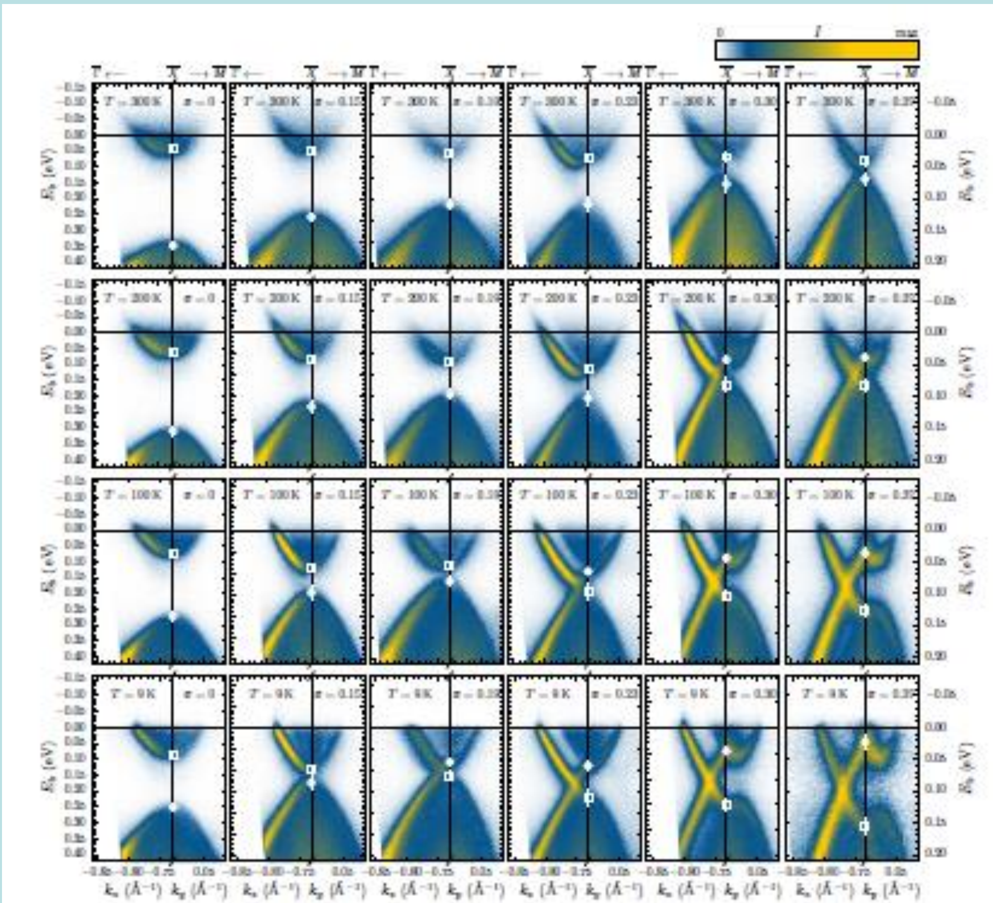
**x=0, 0.15, 0.19, 0.23, 0.30, 0.37**

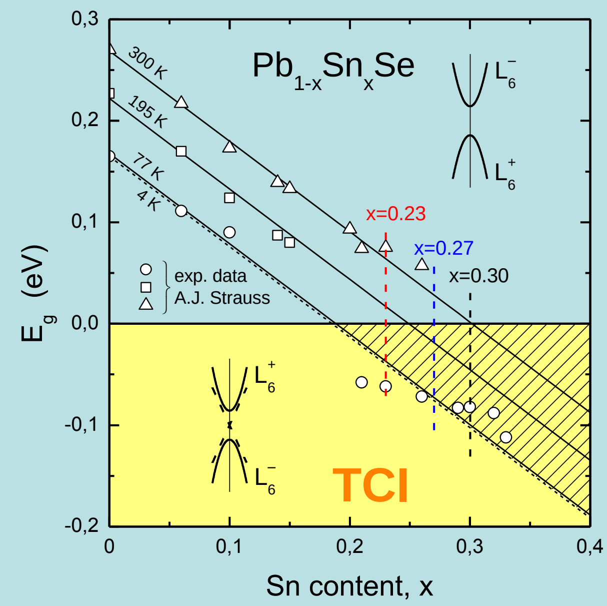
**T=300 K**

**T= 200 K**

**T=100 K**

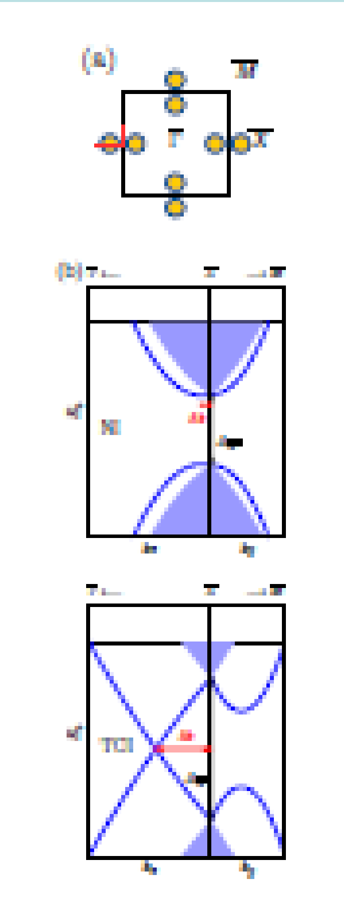
**T=9 K**

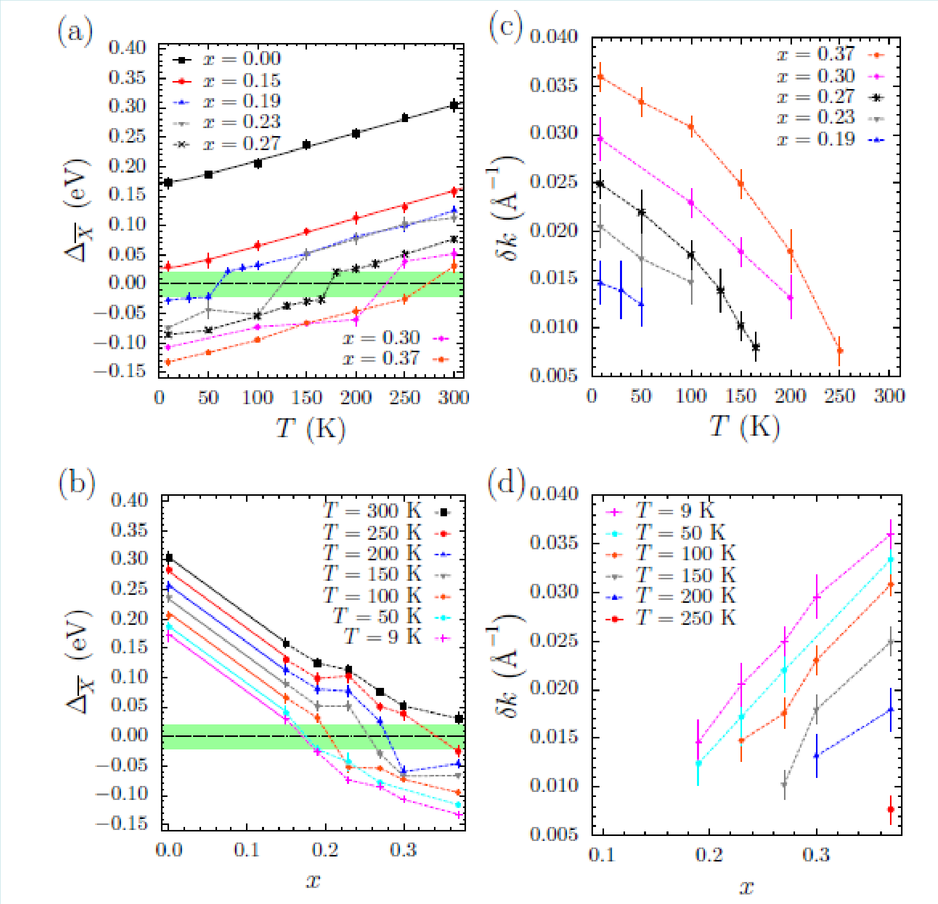





**B.M. Wojek , P. Dziawa, B.J. Kowalski, A. Szczerbakow, A.M. Black-Schaffer, M.H. Berntsen, T. Balasubramanian, T. Story, O. Tjernberg, Phys. Rev. B 90 161202 (2014)**

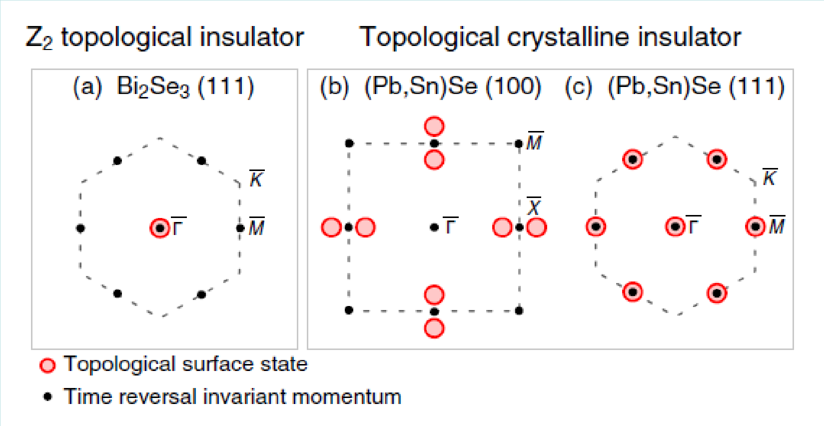
# $Pb_{1-x}Sn_xSe$: Topological T-x phase diagram

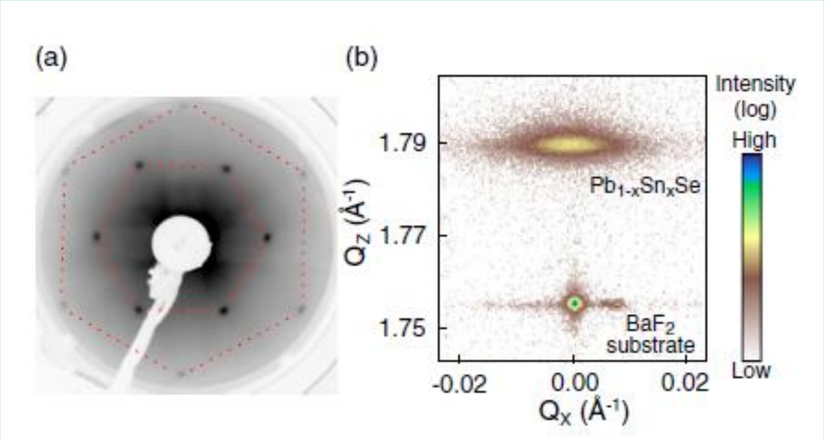


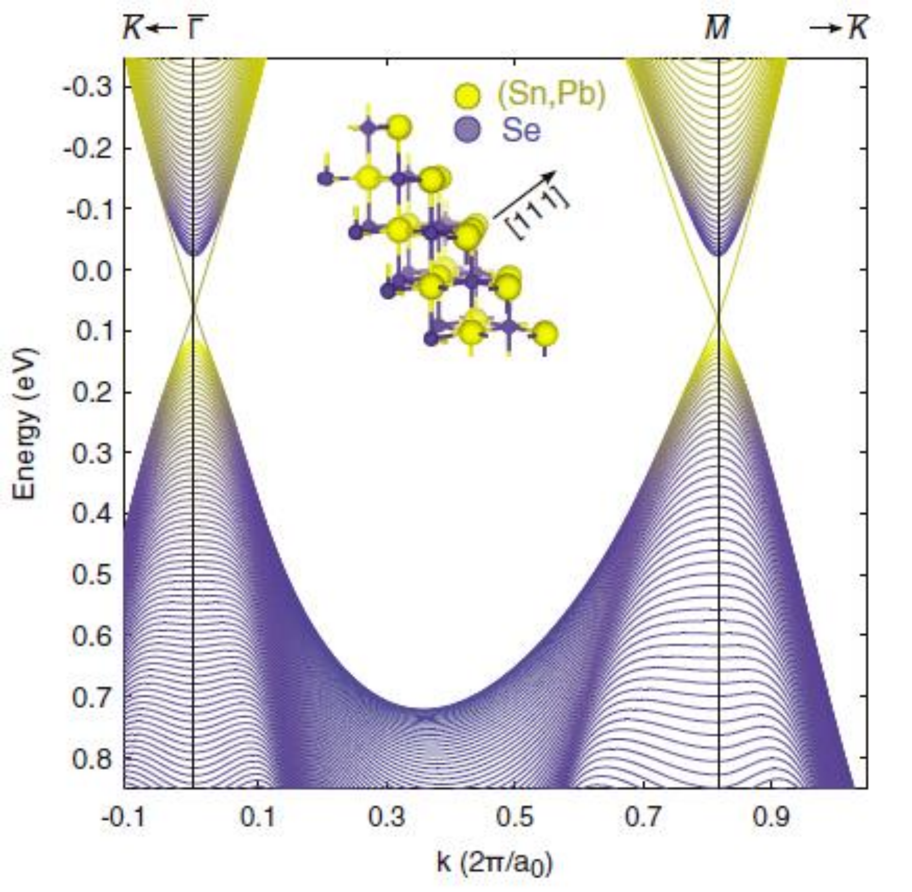



B.M. Wojek , P. Dziawa, B.J. Kowalski, A. Szczerbakow, A.M. Black-Schaffer, M.H. Berntsen, T. Balasubramanian, T. Story, O. Tjernberg, Phys. Rev. B Rapid Com . .90, 161202 (2014)

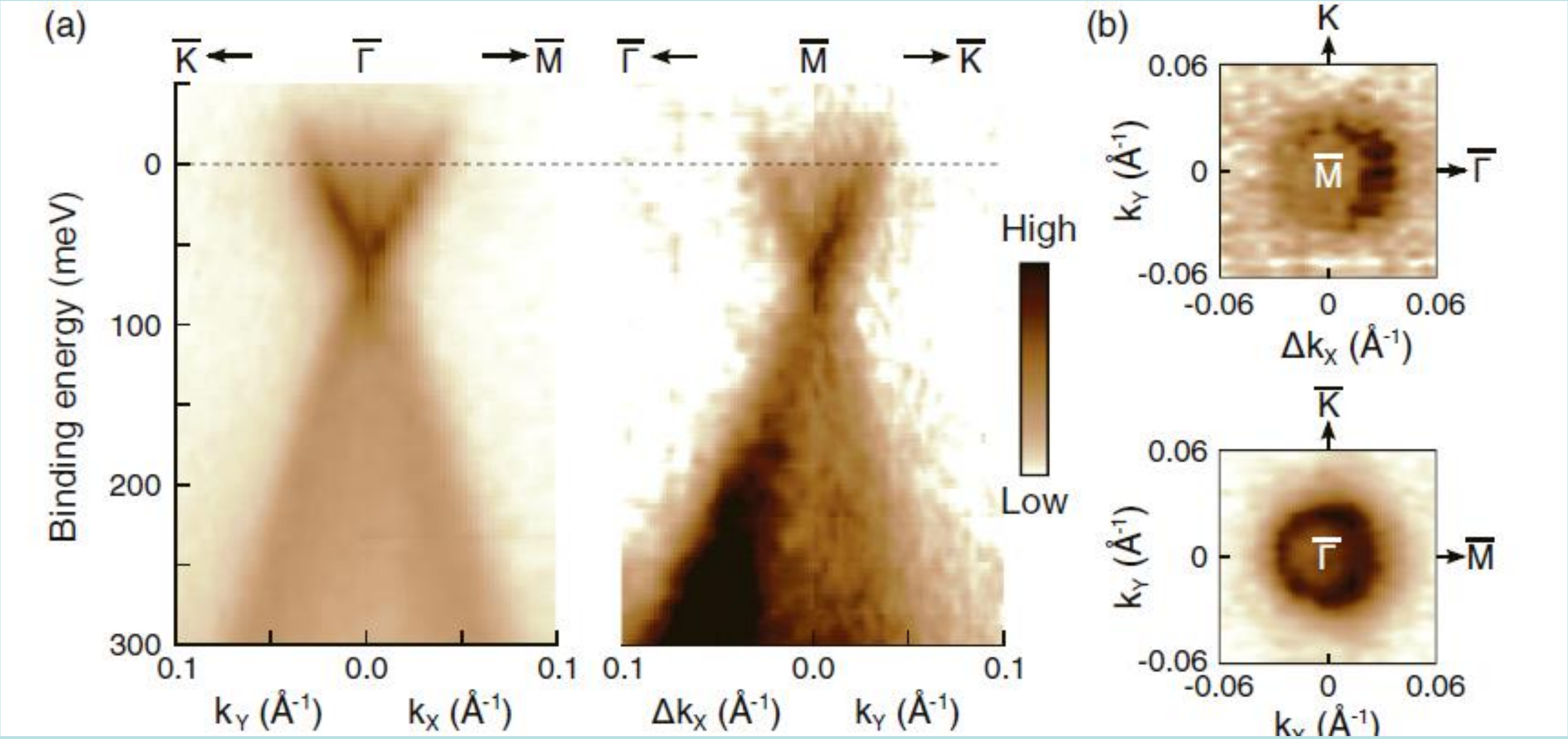
# $Pb_{1-x}Sn_xSe/BaF_2$ (111) – thin layers



# $Pb_{1-x}Sn_xSe/BaF_2$ (111) layers

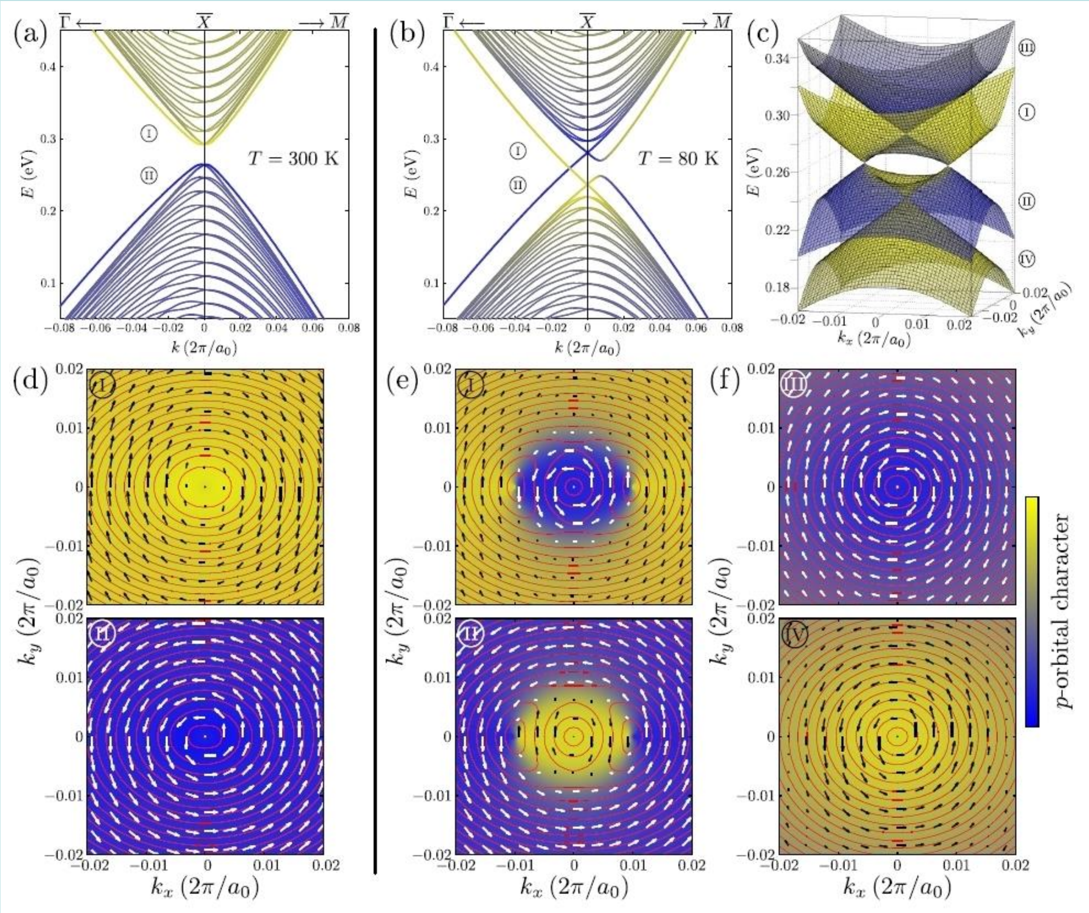



PHYSICAL REVIEW B **89**, 075317 (2014)

**Observation of topological crystalline insulator surface states on (111)-oriented $Pb_{1-x}Sn_xSe$ films**

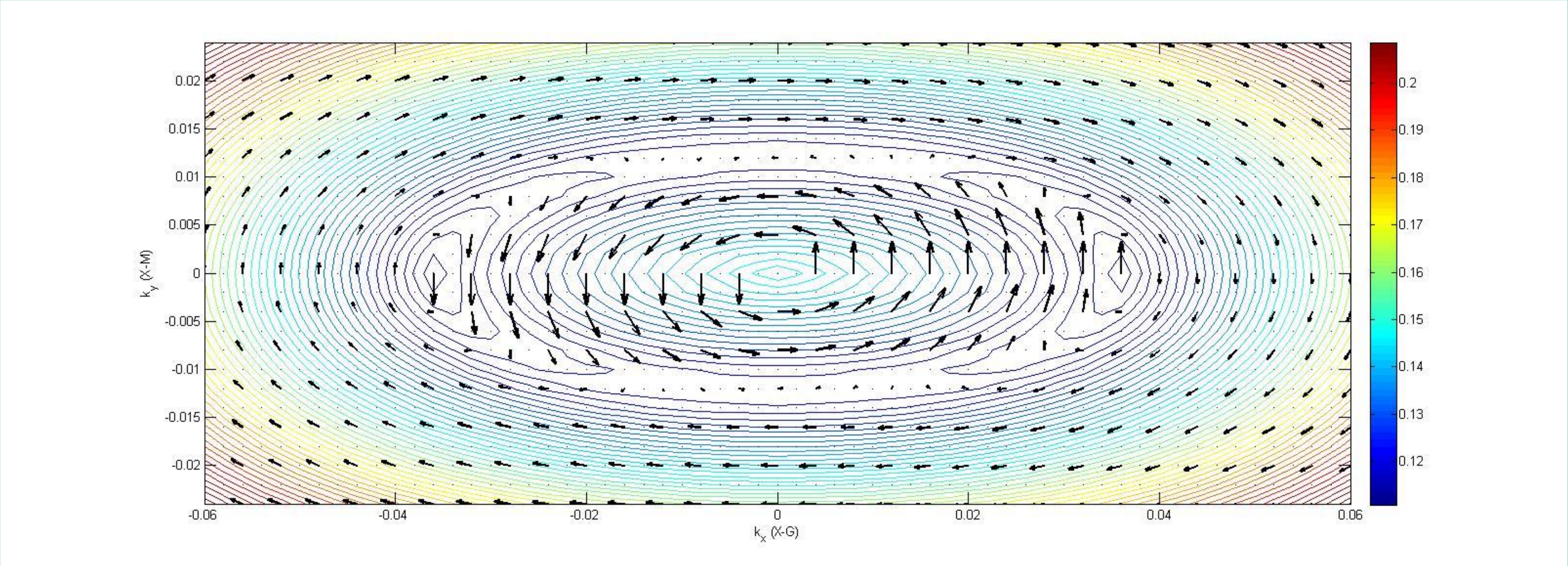
C. M. Polley,[1,*] P. Dziawa,[2] A. Reszka,[2] A. Szczerbakow,[2] R. Minikayev,[2] J. Z. Domagala,[2] S. Safaei,[2] P. Kacman,[2] R. Buczko,[2] J. Adell,[1] M. H. Berntsen,[3,†] B. M. Wojek,[3] O. Tjernberg,[3] B. J. Kowalski,[2] T. Story,[2] and T. Balasubramanian[1]

# Spin polarization of TCI states: tight binding model – $Pb_{0.76}Sn_{0.24}Se$



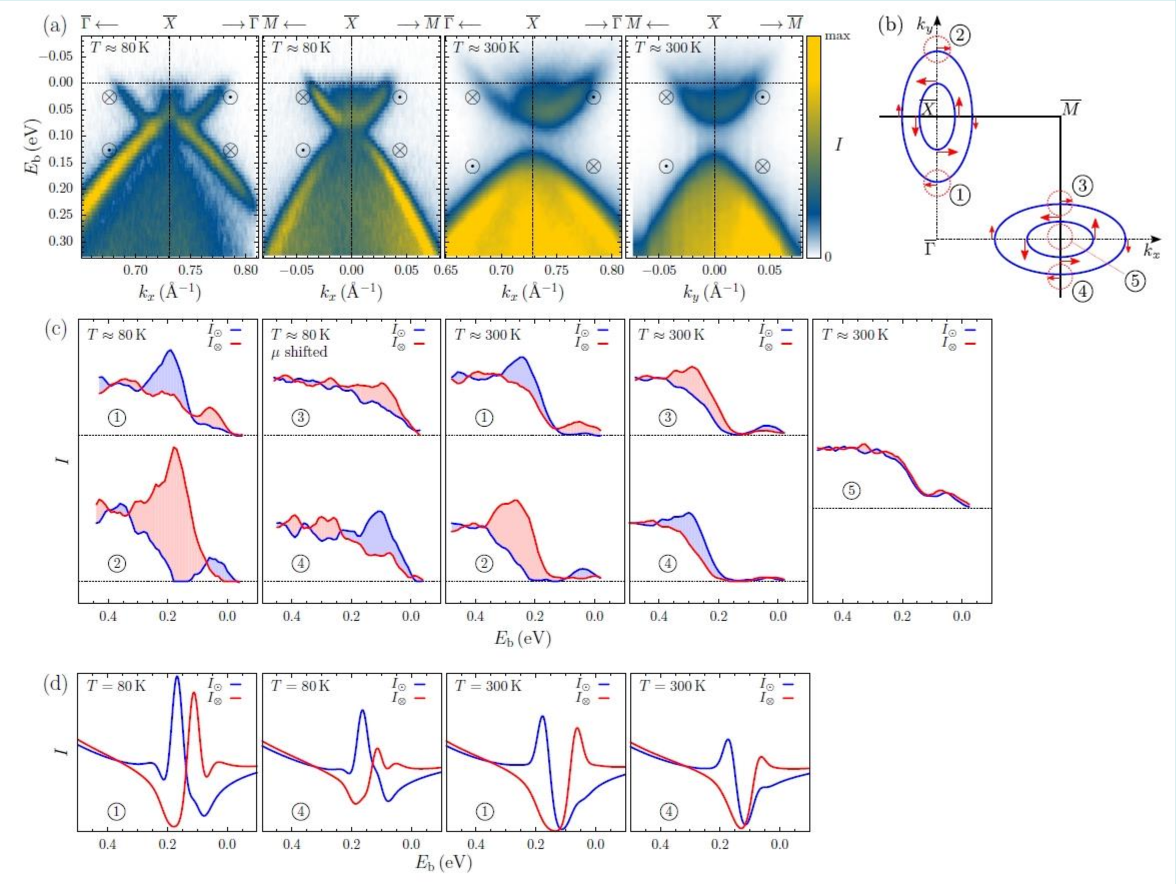

B.M. Wojek, R. Buczko et al. Phys. Rev. B 87, 115105 (2013)

# Spin polarization of TCI states in SnTe



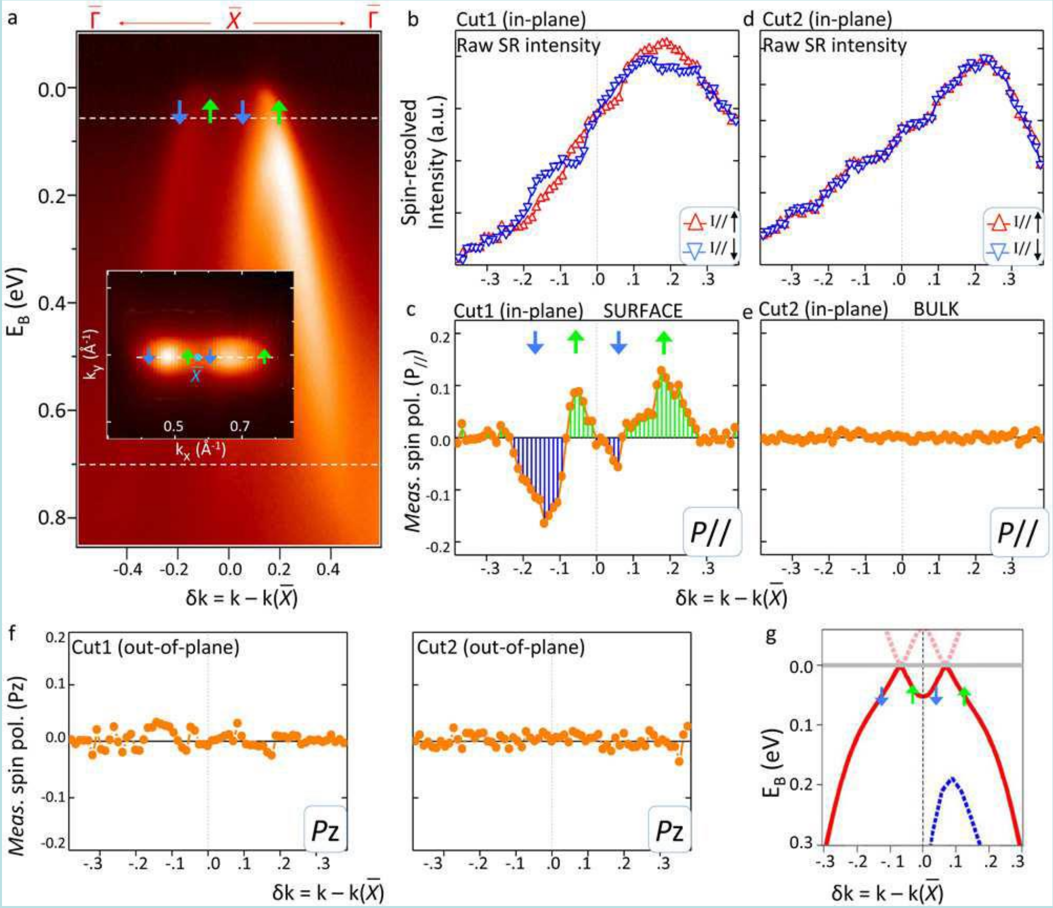

**S. Safaei, P. Kacman, R. Buczko, Phys. Rev. B <u>88</u>, 045305 (2013)**
**tight binding calculations**

# Spin polarization of TCI states: SRPES experiment – $Pb_{0.76}Sn_{0.24}Se$



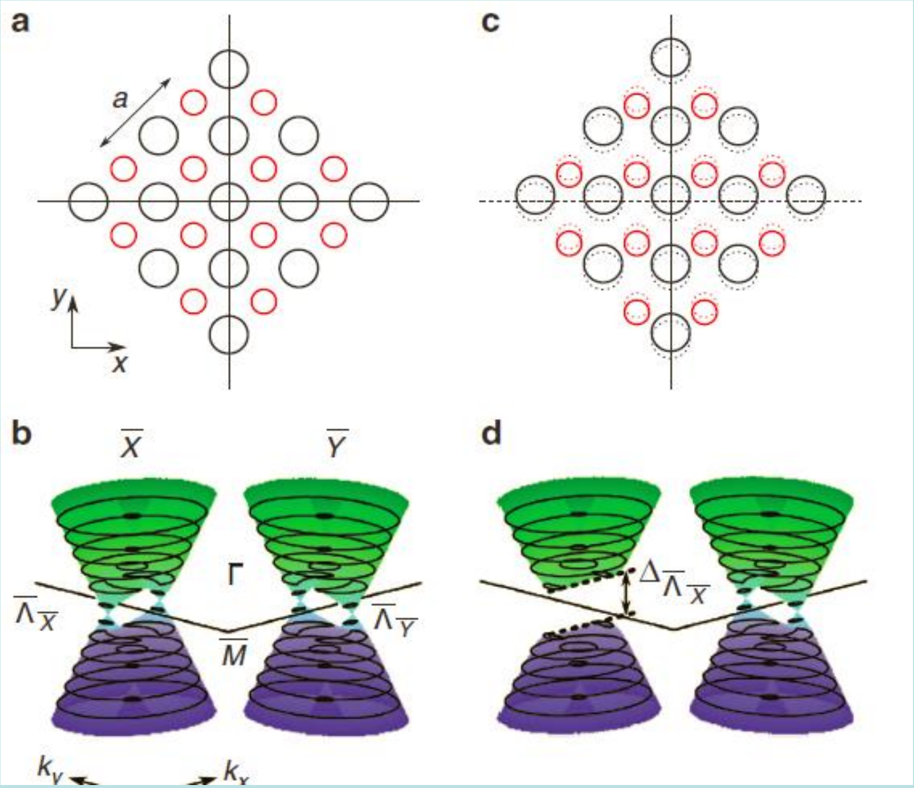

B.M. Wojek, R. Buczko et al.  Phys. Rev. B 87, 115106 (2013)

# Spin polarization in TCI: SRPES – $Pb_{0.6}Sn_{0.4}Te$




- **S-Y Xu, … M.Z. Hasan, Nat. Commun. 3, 1192 (2012).**

# Topological surface states in distorted PbSnSe crystal lattice




ARTICLE

Received 13 May 2015 | Accepted 23 Aug 2015 | Published 13 Oct 2015

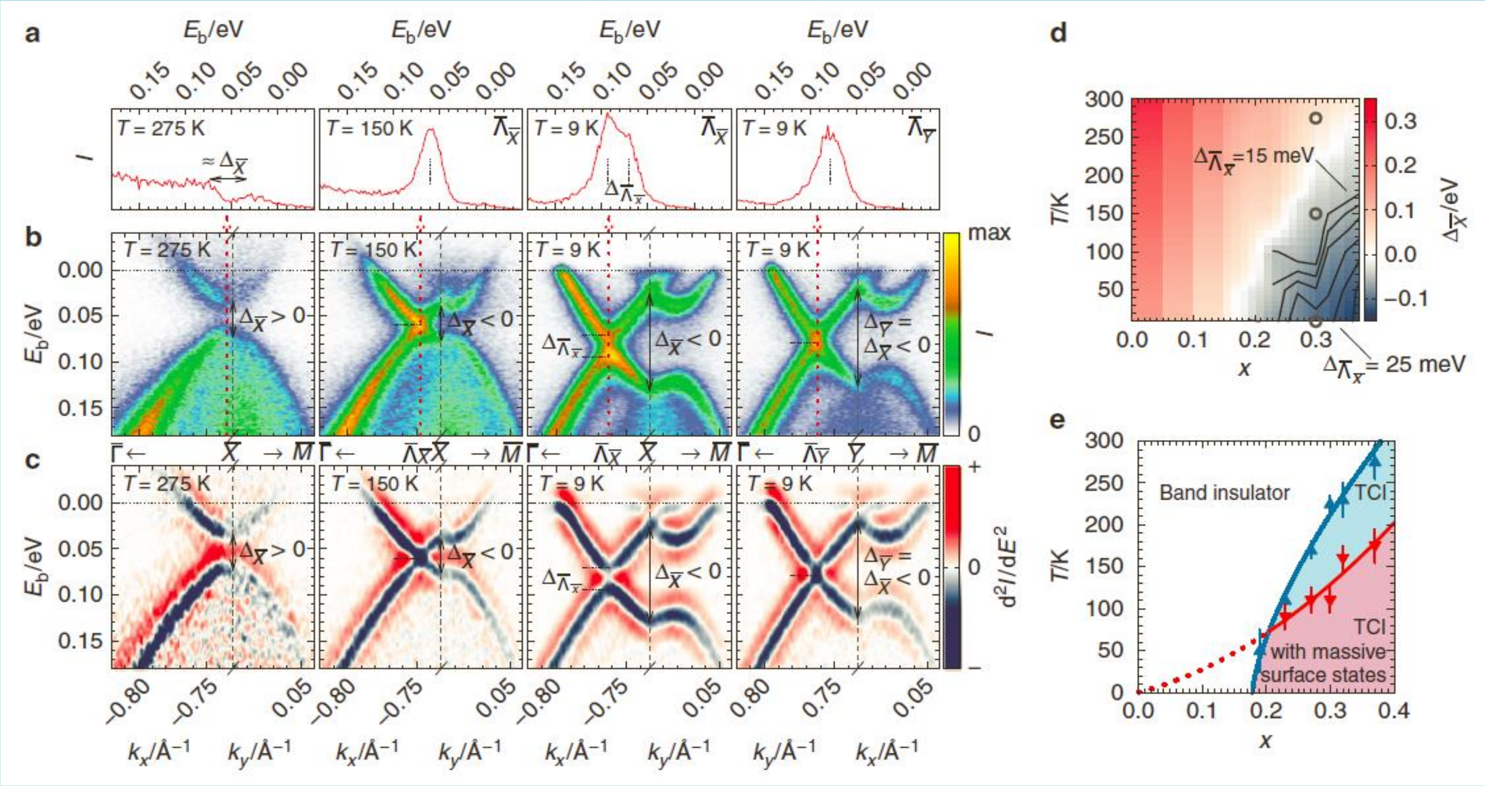
DOI: 10.1038/ncomms9463

OPEN

## Direct observation and temperature control of the surface Dirac gap in a topological crystalline insulator

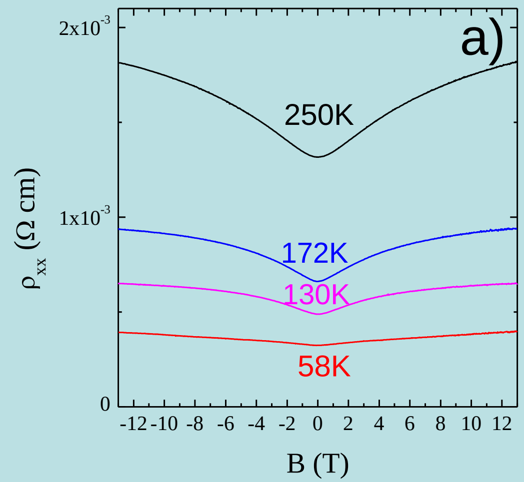
B.M. Wojek[1], M.H. Berntsen[1], V. Jonsson[1,2], A. Szczerbakow[3], P. Dziawa[3], B.J. Kowalski[3], T. Story[3] & O. Tjernberg[1,2]

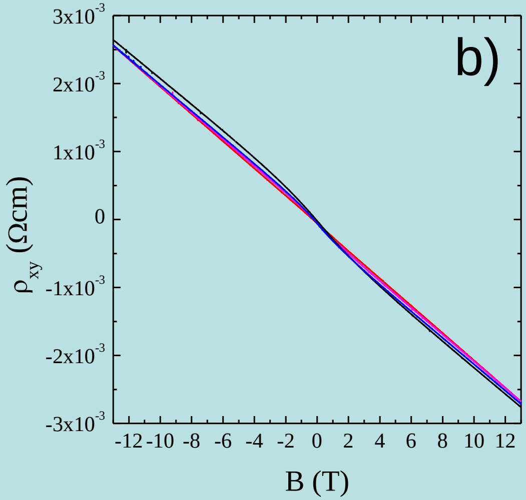
# Topological surface states in distorted PbSnSe crystal lattice

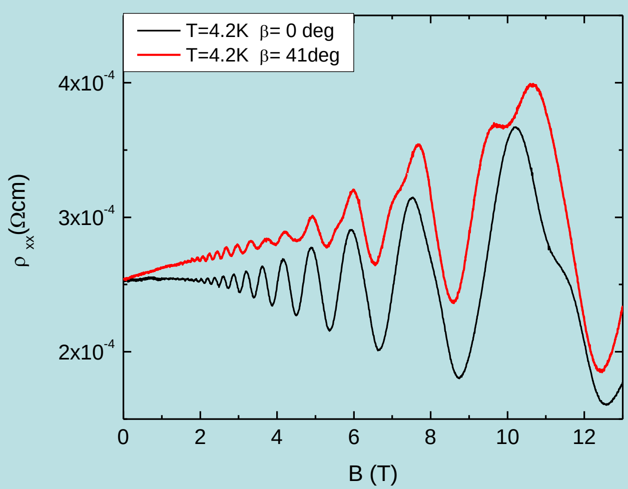


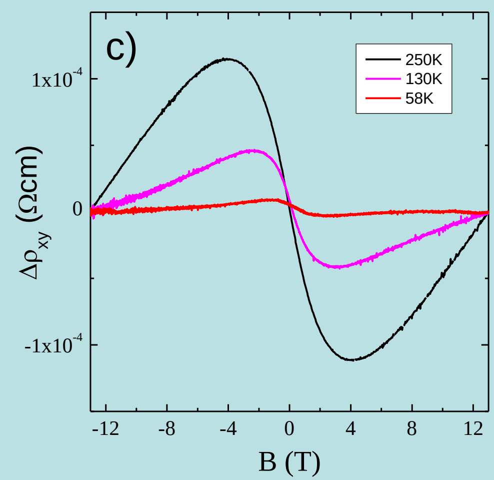

B.M. Wojek et al., Nat. Commun. 6, 8463 (2015)

# $Pb_{0.77}Sn_{0.23}Se$: magneto-transport

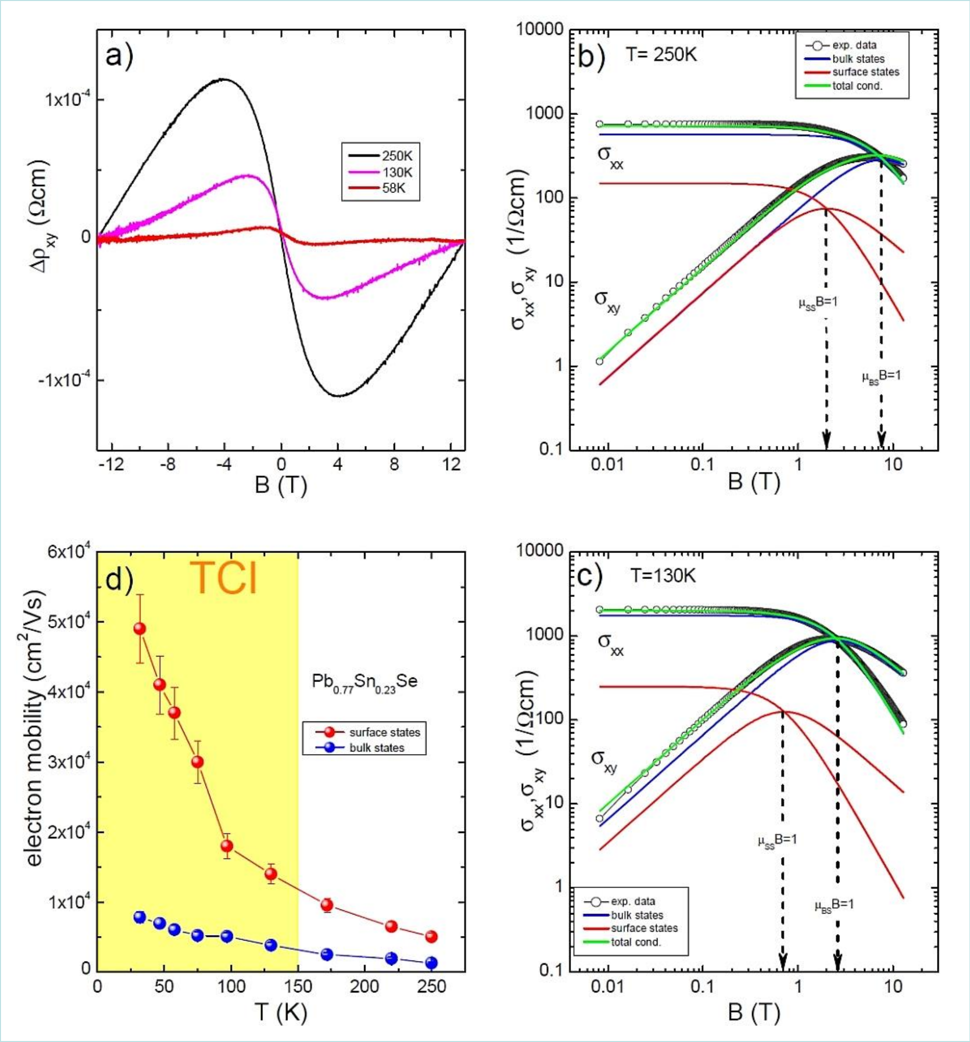


# $Pb_{0.77}Sn_{0.23}Se$: magneto-transport




**Drude model for magneto-conductivity**

**Two parrallel conduction channels**

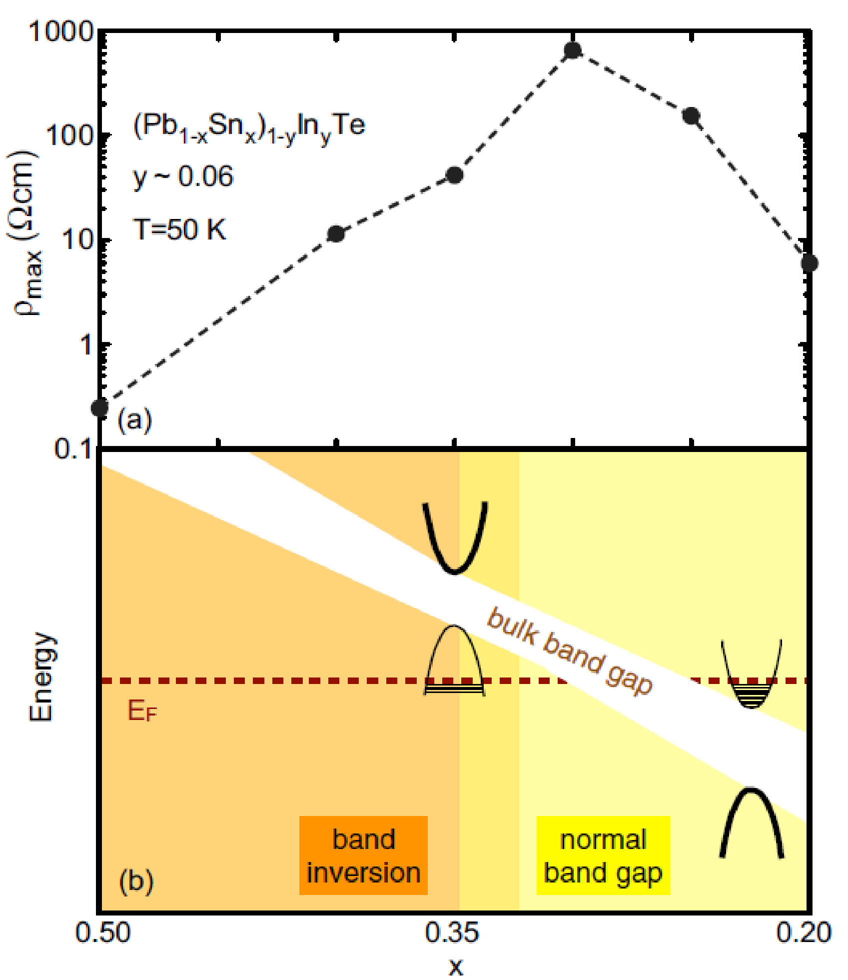
**Fitting of electron transport parameters**

**$\sigma_{BS}$ , $\mu_{BS}$ , $\sigma_{SS}$ , $\mu_{SS}$**

**for bulk crystal and surface channels**

**K. Dybko et al.**

# Controlling electrical properties of TCI materials



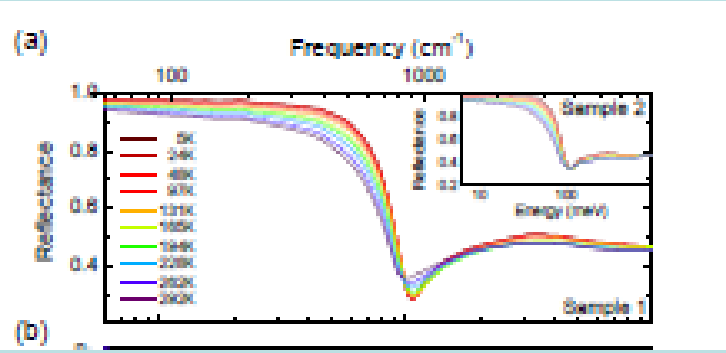

**Deep mid-gap doping centers:**

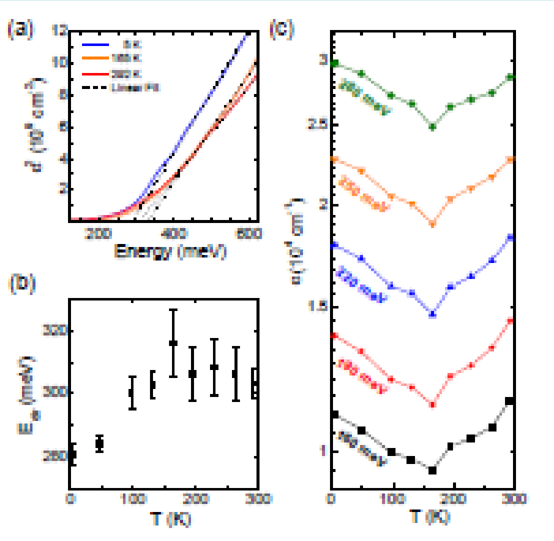
**group III: In**

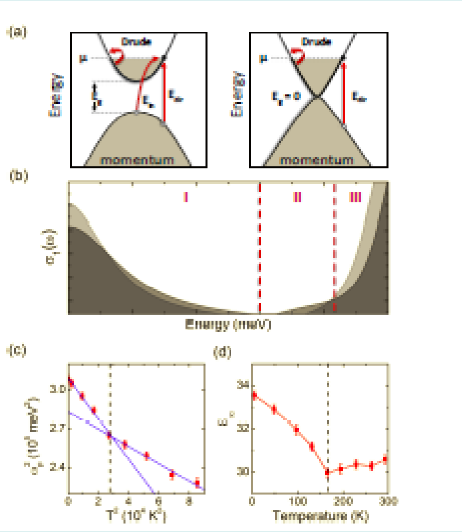
**Transition metals (V, Mo)**

R. Zhong et al., Phys. Rev. B 91, 195321 (2015)

# $Pb_{1-x}Sn_xSe$: infrared optical studies

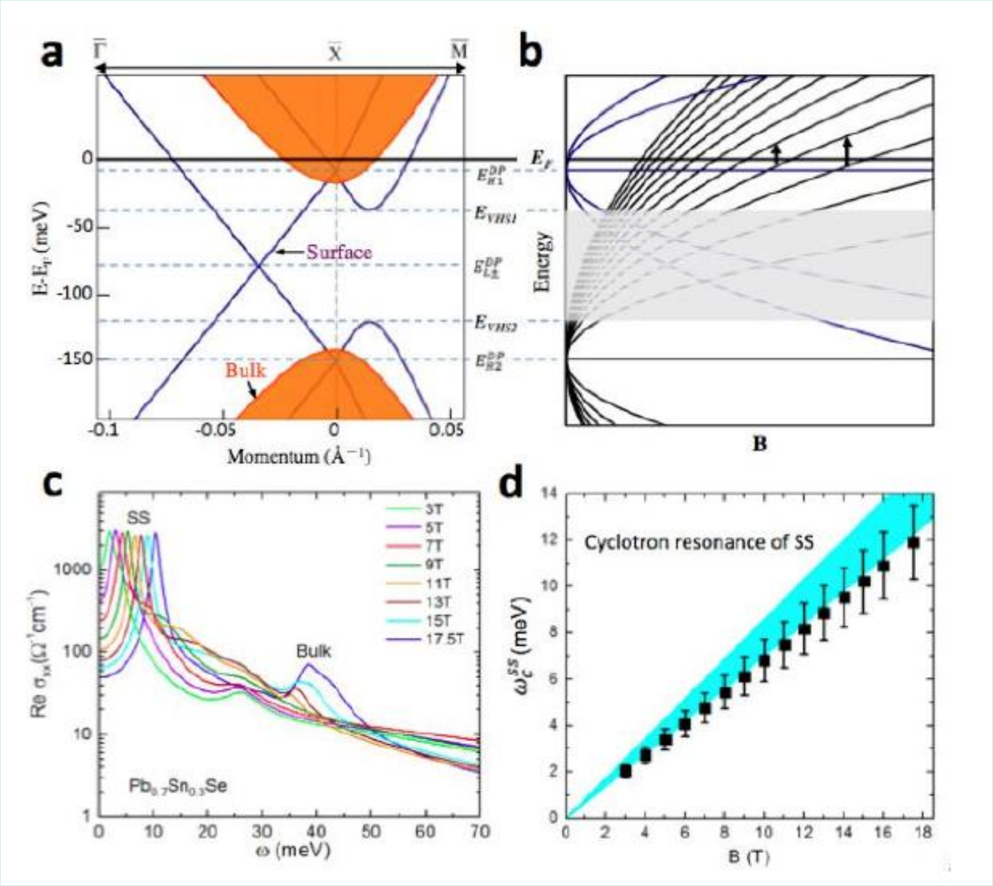



A. Reijnders et al., Phys. Rev B 90, 235144 (2014)
N. Anand et al., Phys. Rev. B 90, 235143 (2014)
X. Xi et al., Phys. Rev. Lett. 113, 096401 (2014)

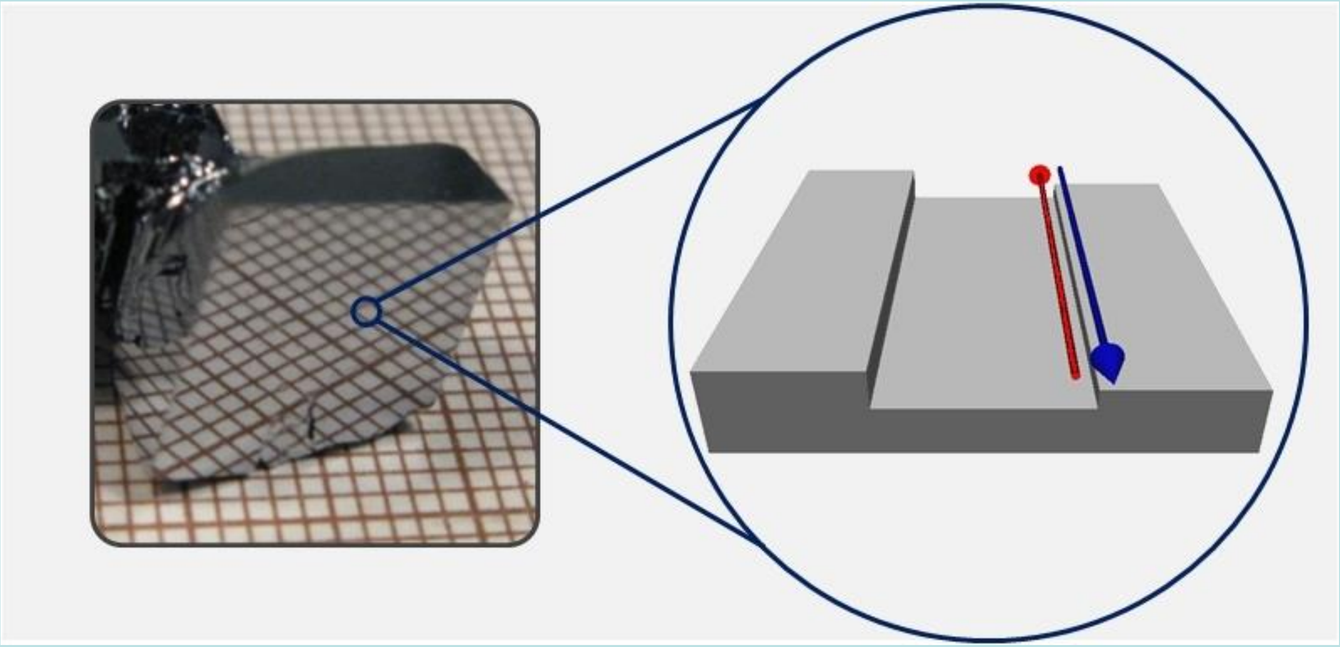
# $Pb_{1-x}Sn_xSe$: infrared magneto-optical studies




Y. Wang et al., arXiv 1611.04302

B.A. Assaf et al., Sci. Rep. 6, 20323 (2016)

# III. Atomic steps as new 1D topological systems



# Atomic steps as new 1D topological systems

RESEARCH | REPORTS

TOPOLOGICAL MATTER

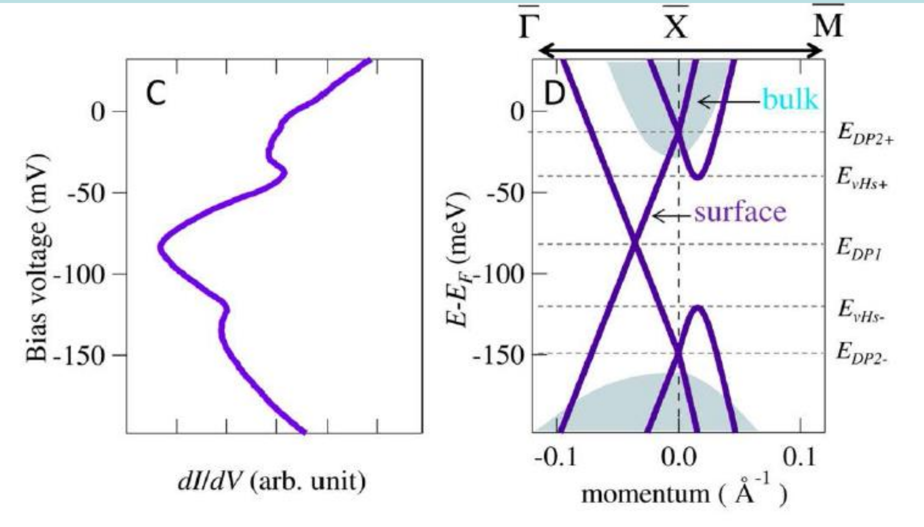
## Robust spin-polarized midgap states at step edges of topological crystalline insulators

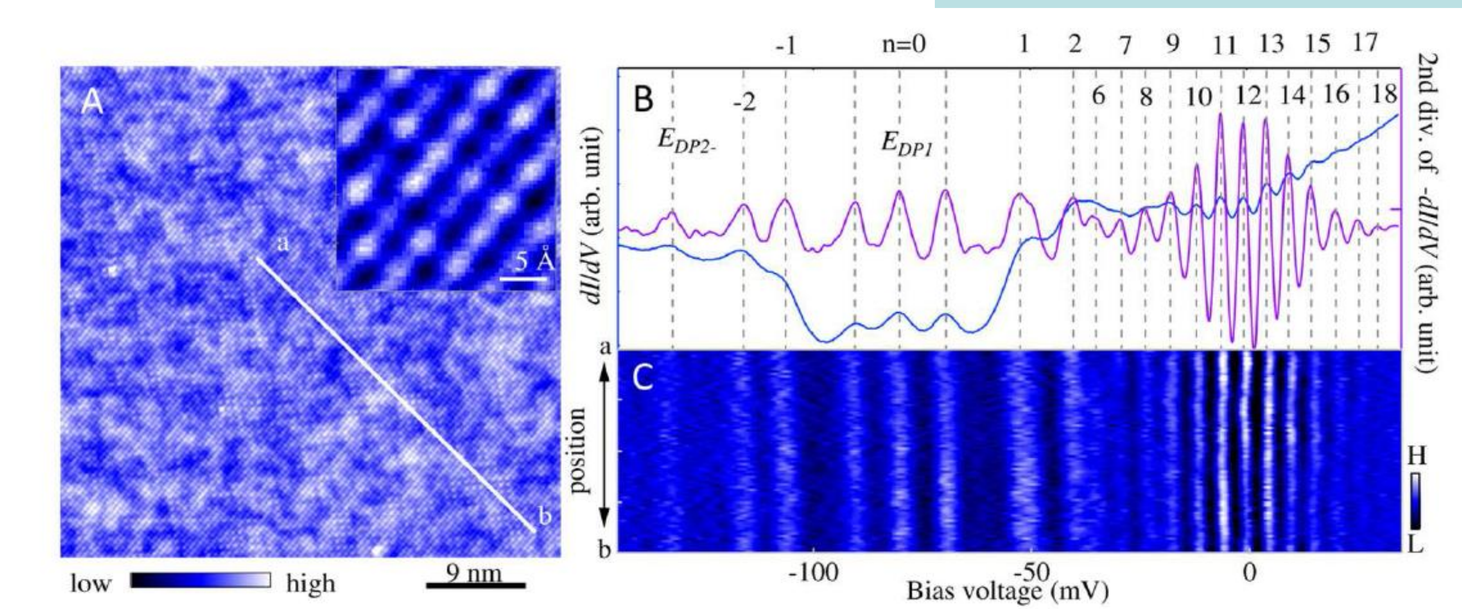
Paolo Sessi,[1*] Domenico Di Sante,[2] Andrzej Szczerbakow,[3] Florian Glott,[1] Stefan Wilfert,[1] Henrik Schmidt,[1] Thomas Bathon,[1] Piotr Dziawa,[3] Martin Greiter,[2] Titus Neupert,[4] Giorgio Sangiovanni,[2] Tomasz Story,[3] Ronny Thomale,[2] Matthias Bode[1,5]

P. Sessi et al., Science 354 (6317), 1269 (2016)

- **Experiment STM/STS**: Würzburg University (EP2)
- **Theory**: Würzburg University (TP1) and Zürich University
- **Monocrystals** (growth and characterization): IP PAS, Warsaw

# STM – conductance spectroscopy

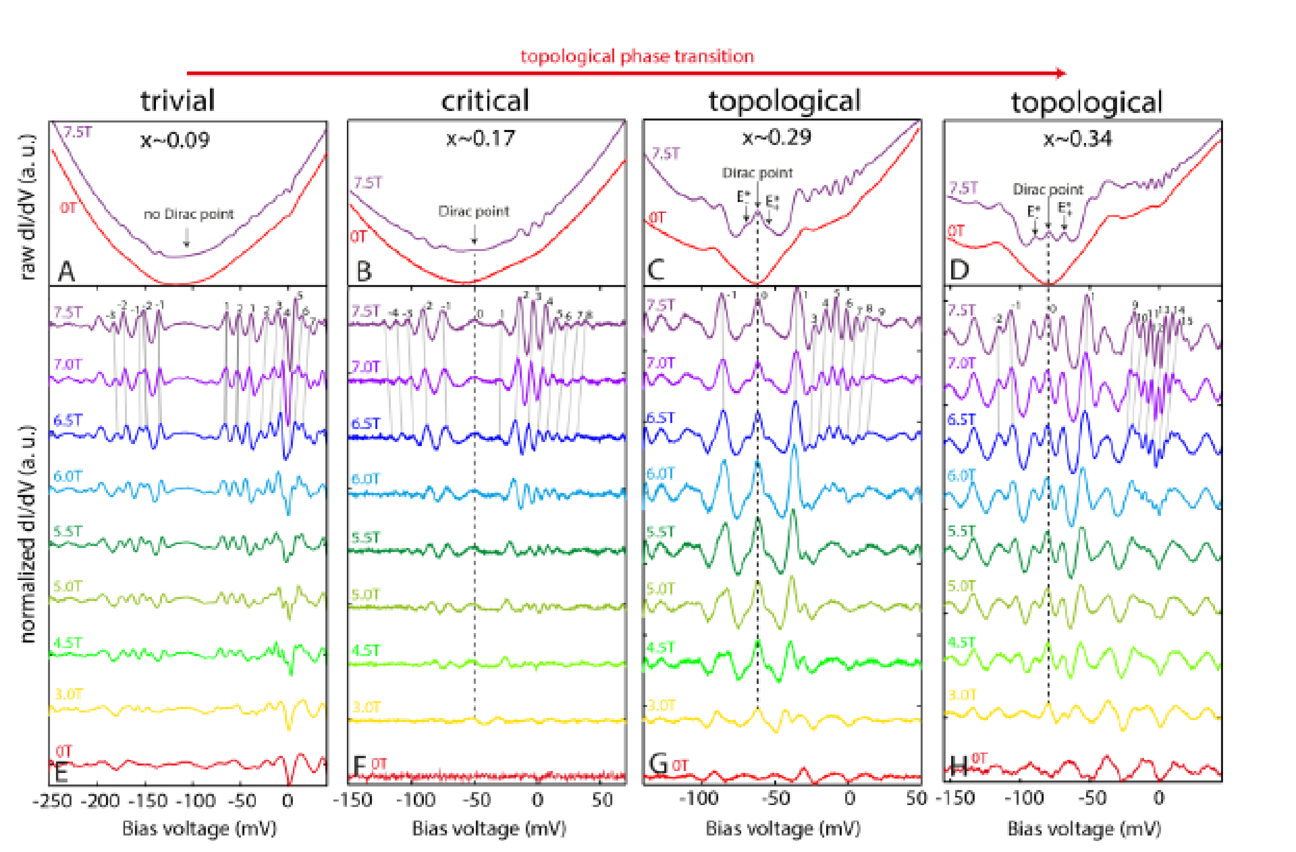



**$Pb_{1-x}Sn_xSe$**

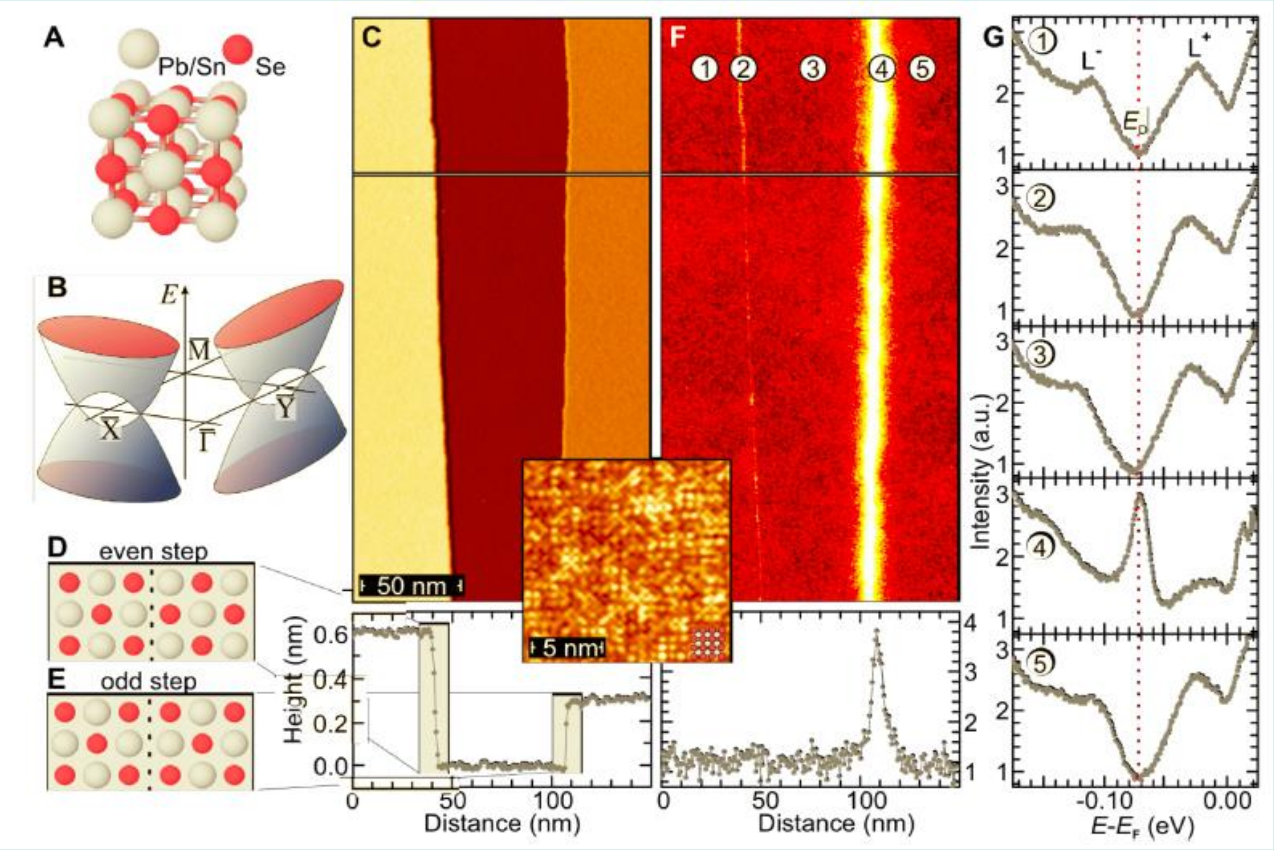
**Y. Okada et al.,**
**Science 341, 1496 (2013)**

# $Pb_{1-x}Sn_xSe$ - STM spectroscopy Landau quantization



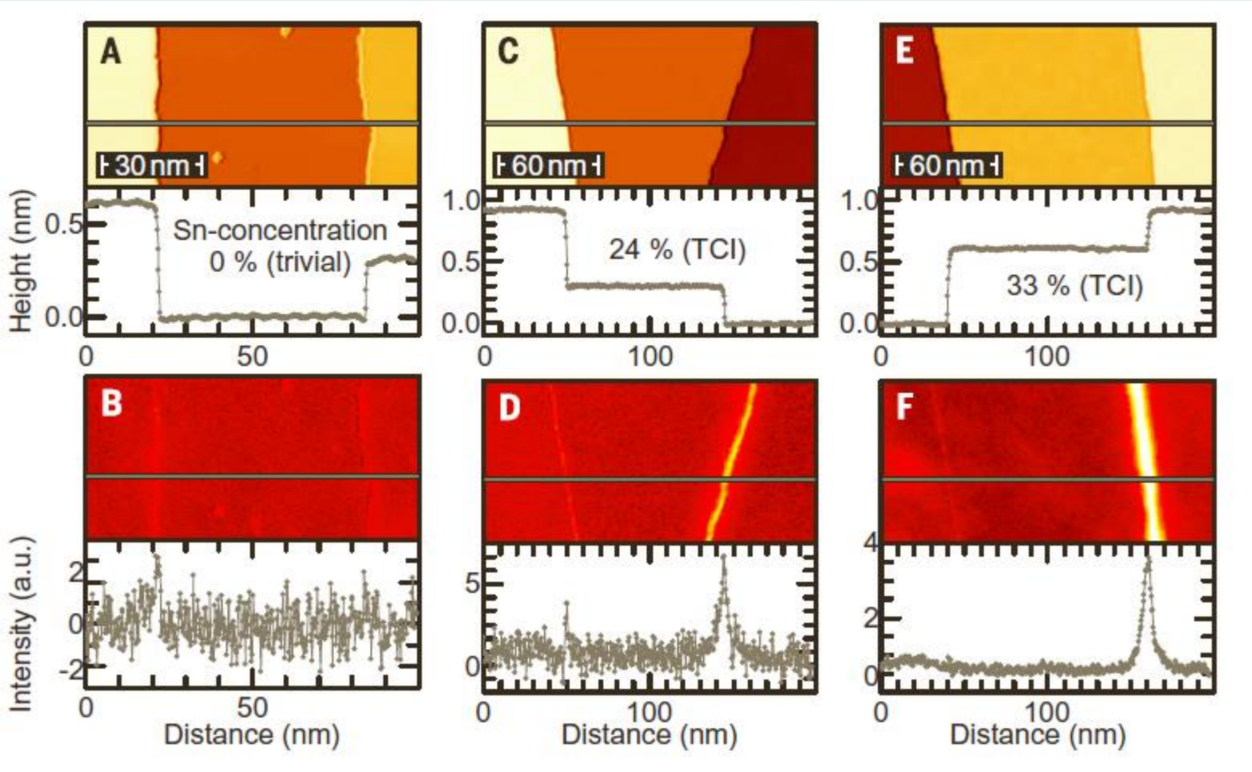

I. Zeljkovic, …V. Madhavan, Nature Physics 10, 572 (2014);

# STM/STS spectroscopy in $Pb_{1-x}Sn_xSe$: topological 1D states



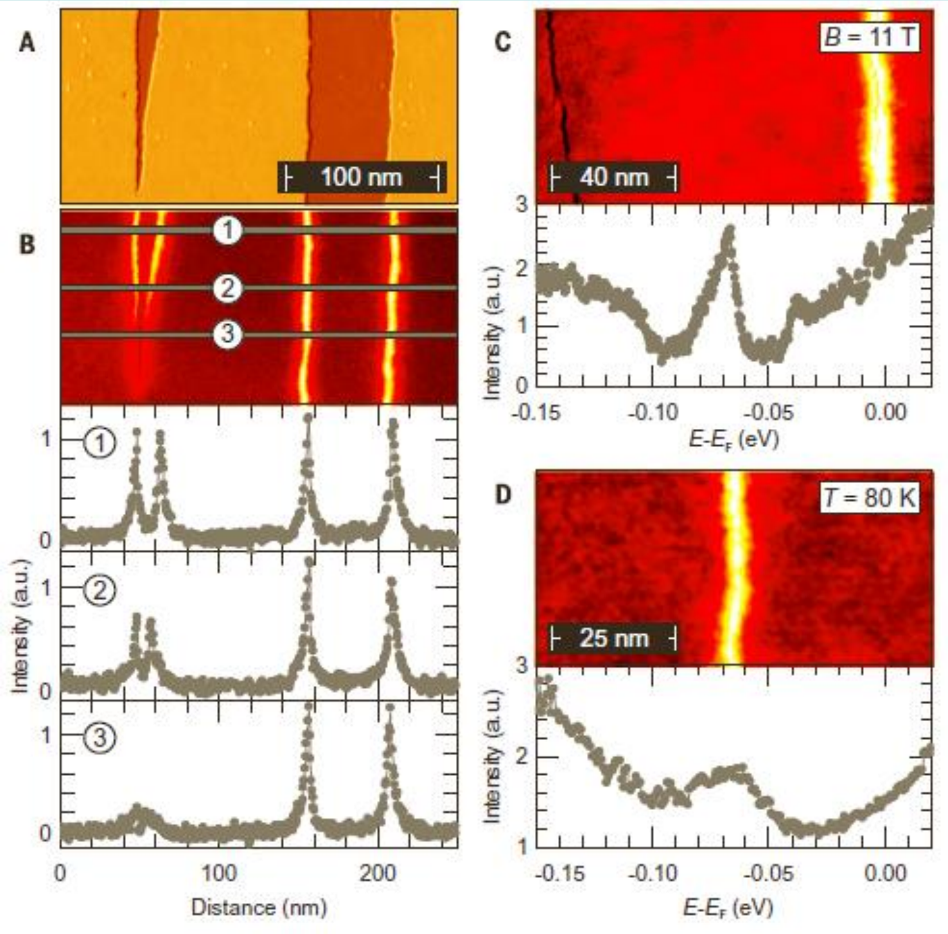

P. Sessi et al., Science 354 (6317), 1269 (2016)

# STM/STS spectroscopy in $Pb_{1-x}Sn_xSe$: topological 1D states



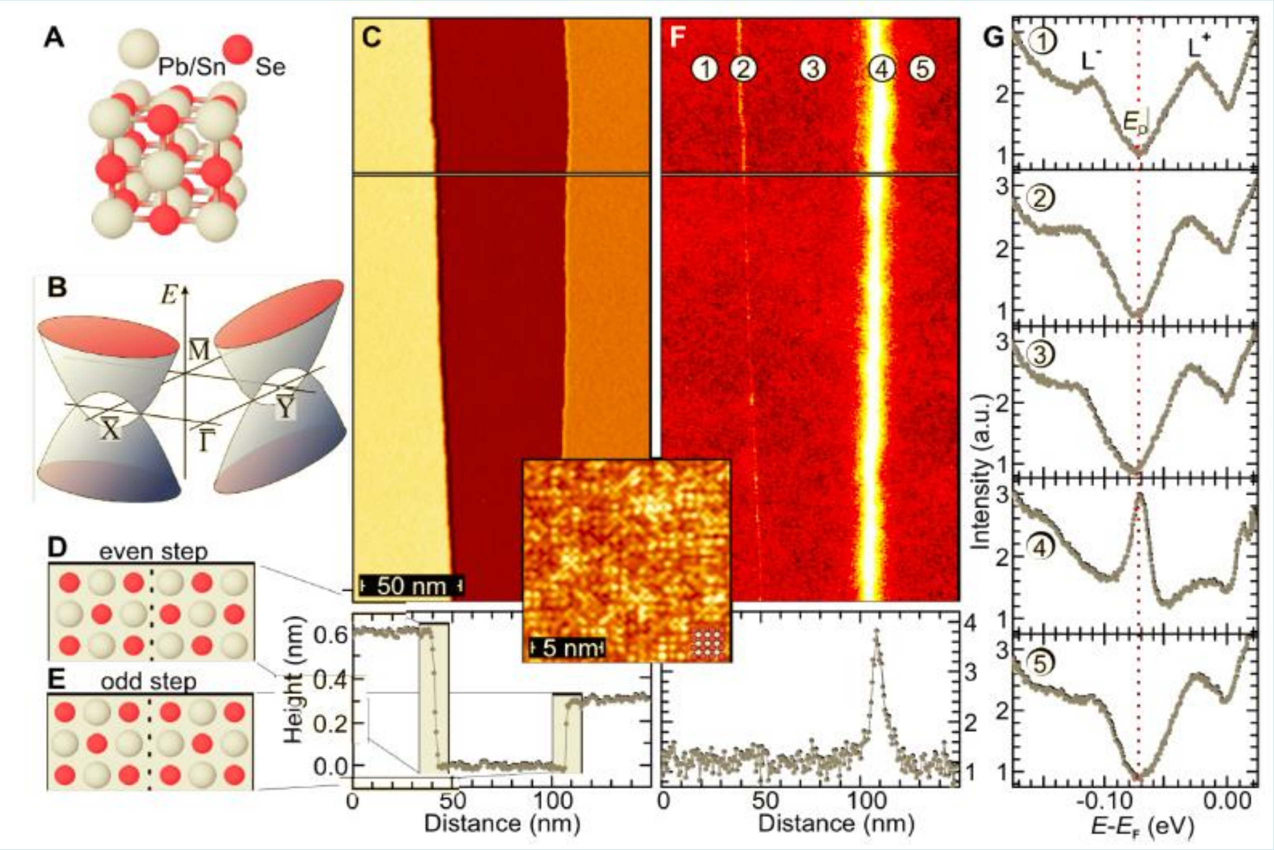

P. Sessi et al., Science <u>354</u> (6317), 1269 (2016)

# STM/STS spectroscopy in $Pb_{1-x}Sn_xSe$: topological 1D states



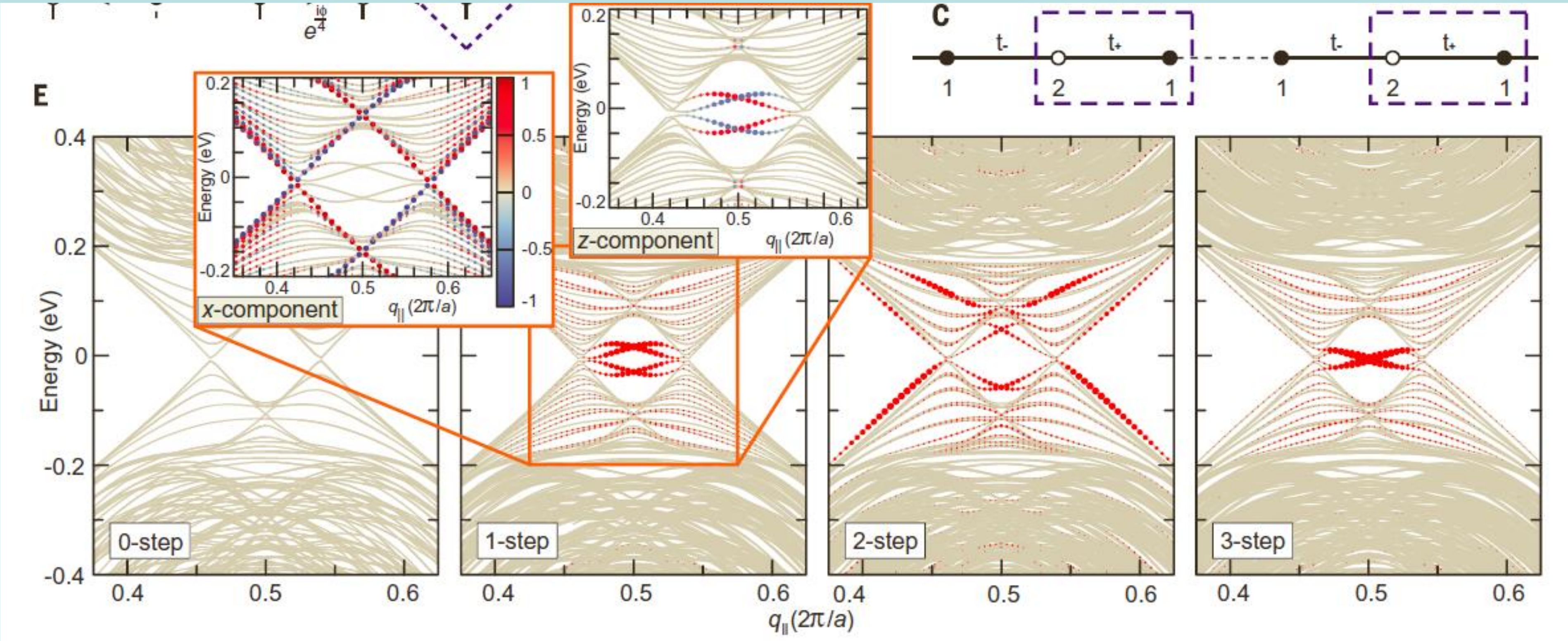

P. Sessi et al., Science 354 (6317), 1269 (2016)

# STM/STS spectroscopy in $Pb_{1-x}Sn_xSe$: topological 1D states



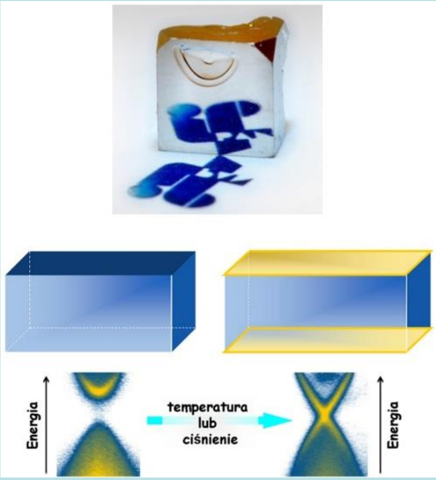

P. Sessi et al., Science 354 (6317), 1269 (2016)

# Electronic structure of $Pb_{1-x}Sn_xSe$ (001) with atomic steps




P. Sessi et al., Science <u>354</u> (6317), 1269 (2016)

# Topological materials - summary




- Metalic  surface (or edge) states with linear energy dispersion (massless Dirac fermions).
- Inverted band structure: relativistic effects (spin-orbit coupling, Darwin term).
- Topological protection, suppression of backscattering.
- Spin polarization, helical states, spin-momentum locking.

- Bandgap origin and symmetry protection based classification:
- topological insulators (TRS),
- topological crystalline insulators (mirror-plane symmetry):

  IV-VI semiconductors SnTe , (Pb,Sn)Te and (Pb,Sn)Se