Ballistic effects in hBN-encapsulated graphene and moiré superlattice minibands in G/hBN heterostructures

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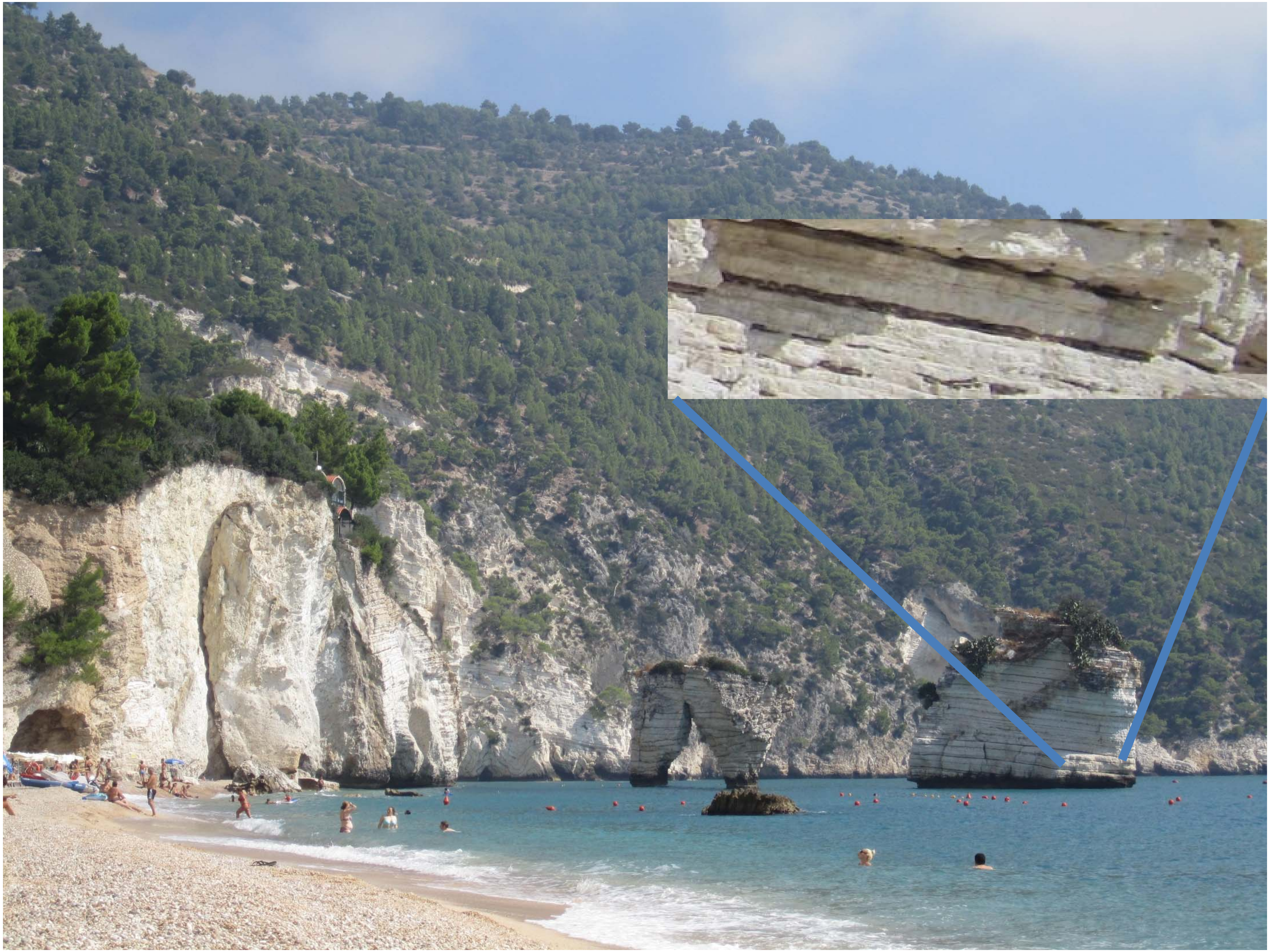
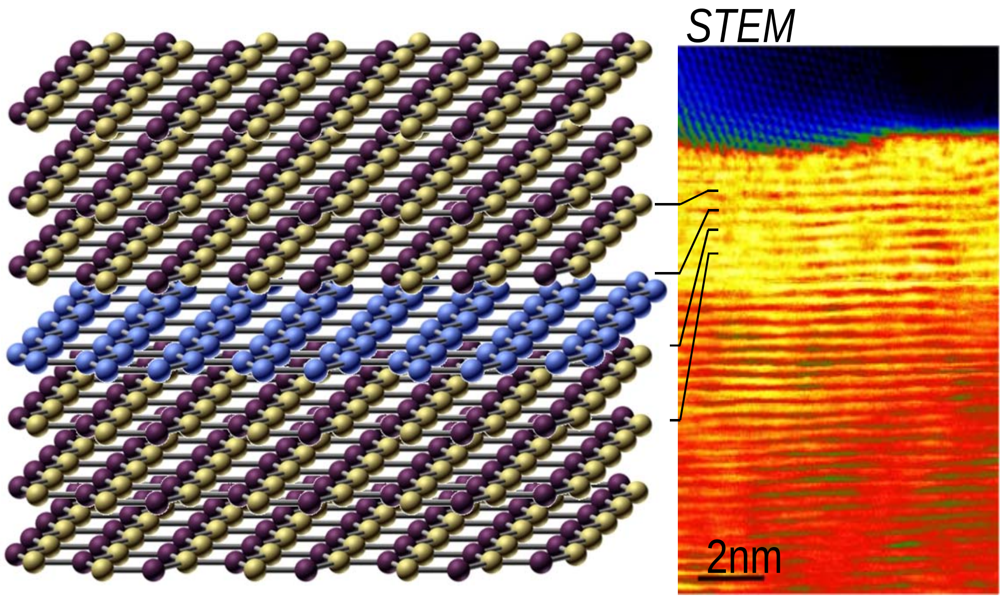
GRAPHENE FLAGSHIP



Graphene: gapless semiconductor with Dirac electrons

$$\hat{H} = v\vec{\sigma}\cdot\vec{p}$$

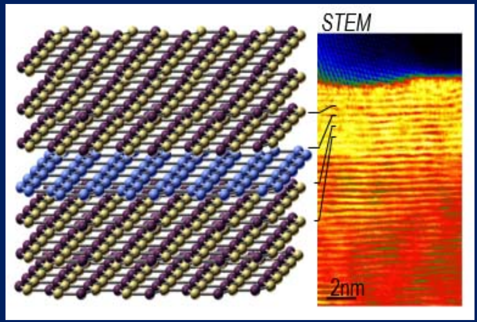


hBN ('white graphene') sp^2 – bonded insulator with a large band gap, Δ >5eV

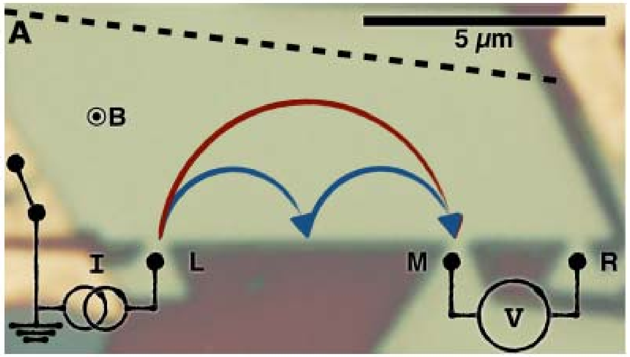
$$\hat{H} = \Delta\sigma_z + v'\vec{\sigma}\cdot\vec{p}$$

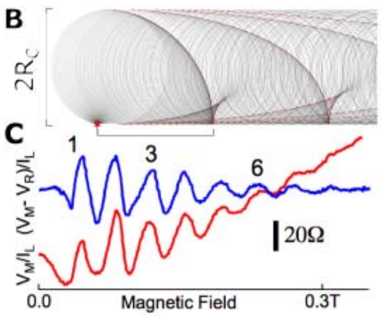
- **Graphene at its best: ballistic electrons in graphene (G) encapsulated in van der Waals heterostructures with hexagonal boron nitride (hBN)**

- **Moiré superlattice in graphene – hBN heterostructures, moiré minibands, and Zak-Brown magnetic minibands**



hBN-encapsulated graphene: multi-μm ballistic transport proven by transverse electron focusing

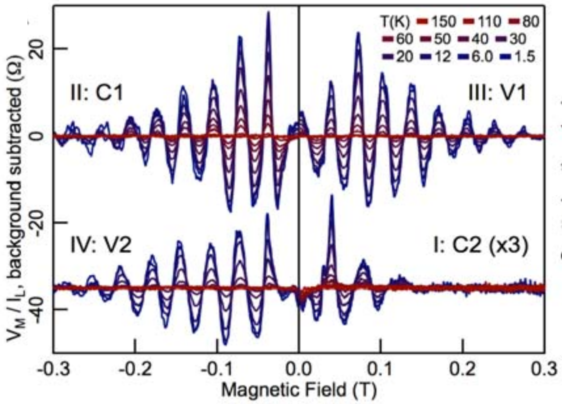


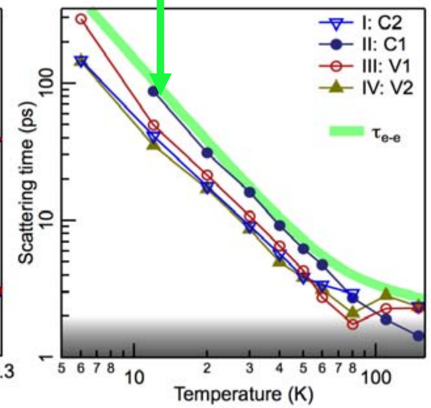


Transverse magnetic focusing (caustics of skipping orbits) of ballistic electrons

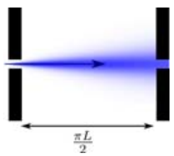
$$\frac{A(T)}{A(T_{base})} \sim e^{-\pi L/2v_F\tau}$$

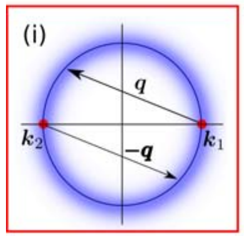
$$\tau\,(T) = -\frac{2v_F}{\pi L}\log\frac{A_1(T)}{A_1(T_{base})}$$





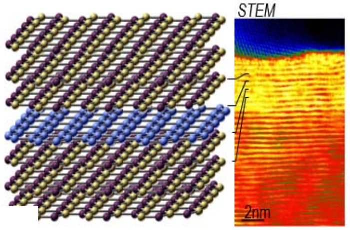
$$[\partial_t + v\sin(\theta_1)\partial_y]\,f(\vec{k}_1) = I\{f(\vec{k}_1)\}$$



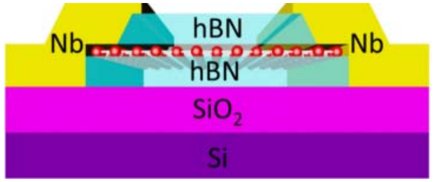


Taychatanapat, Watanabe, Taniguchi, Jarillo-Herrero - Nature Phys 9, 225 (2013)

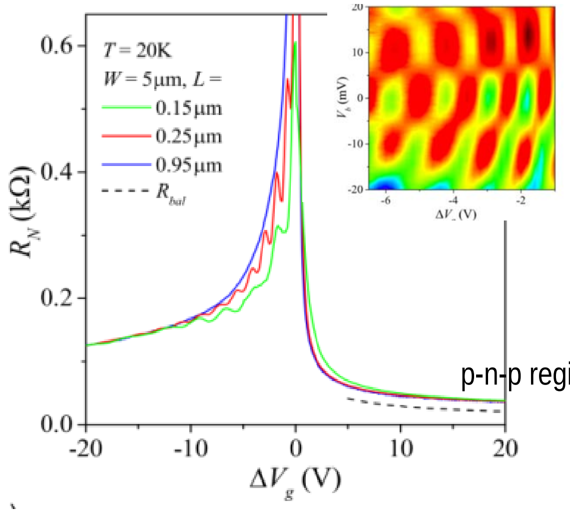
Lee, Wallbank, Gallagher, Watanabe, Taniguchi, Fal'ko, Goldhaber-Gordon - Science 353, 1526 (2016)



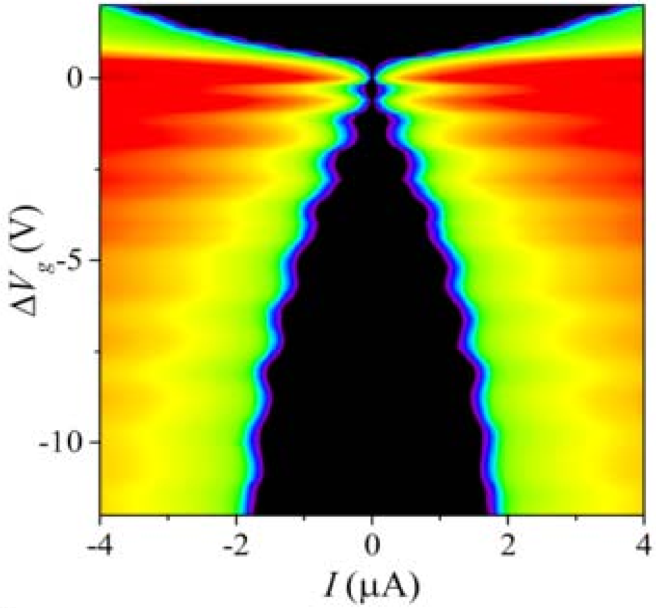
Fabry-Perot oscillations of I(V) and critical supercurrent in hBN/G/hBN with S-leads



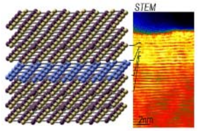
Ballistic SNS:
Fabry-Perot oscillations of critical supercurrent current



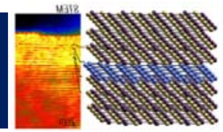
Ballistic graphene:
Fabry-Perot oscillations of ***dI/dV***

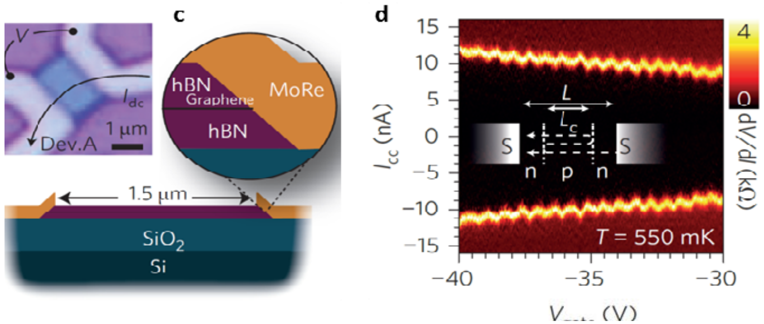


Ben-Shalom, Zhu, Fal'ko, Mishchenko, Kretinin, Novoselov, Woods, Watanabe, Taniguchi, Geim, Prance
Nature Physics 12, 318 (2015)

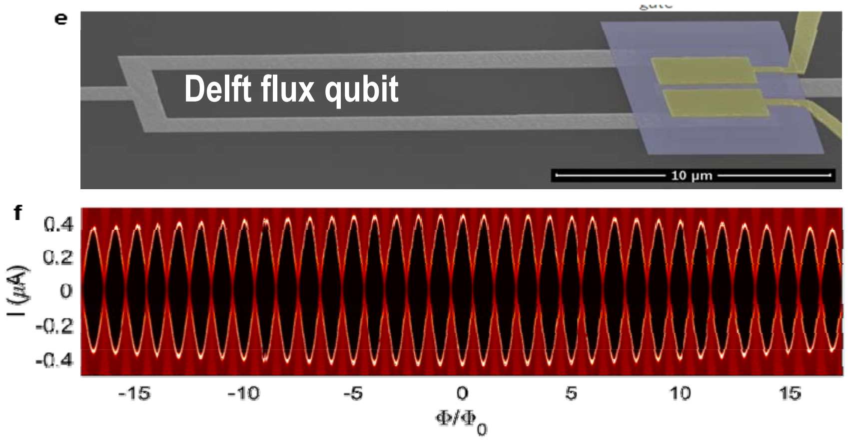


QT devices using ballistic SGS

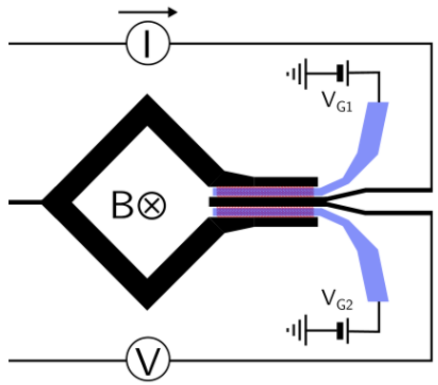


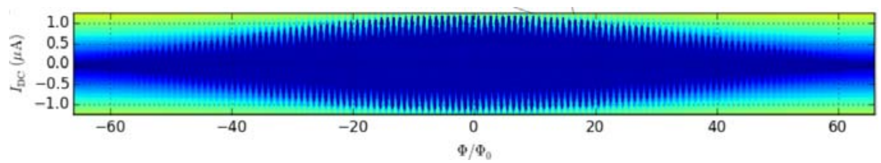


Calado, Goswami, Nanda, Diez, Akhmerov, Watanabe, Taniguchi, Klapwijk, Vandersypen Nature Nanotechnology 10, 761 (2015)



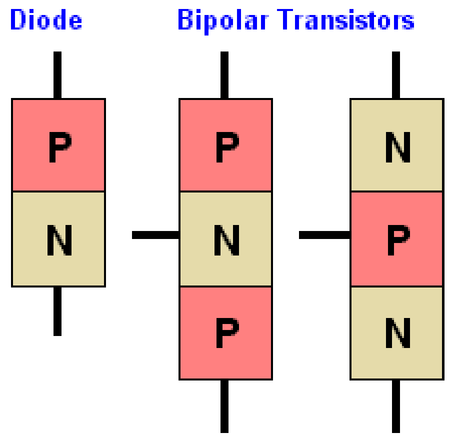
Lancaster graphene FET-based SQUID: supercurrent can be switched on/off fast using electrostatic gates:





quantum device for magnetic field measurement

PN junctions

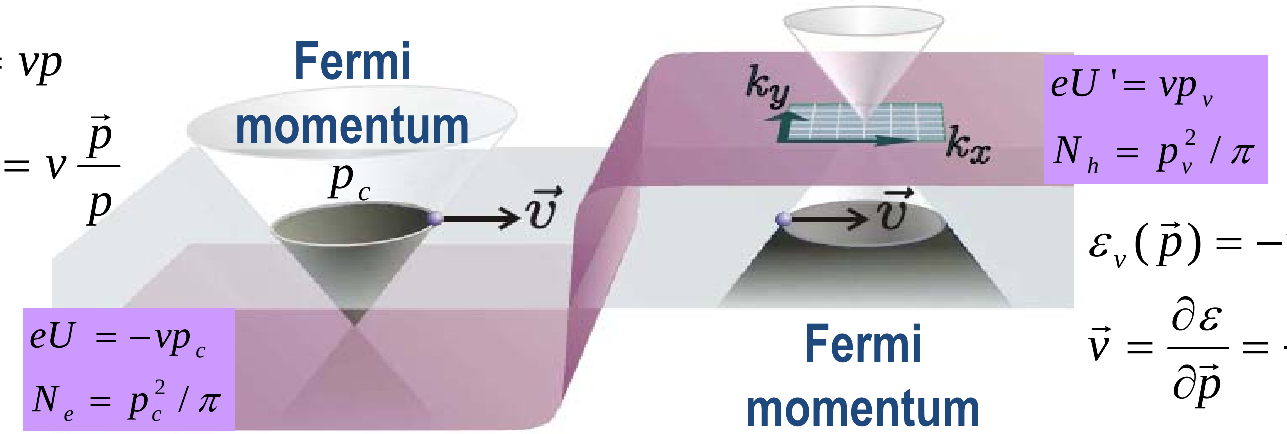


Tunneling PN junctions in semiconductors

Ballistic PN junction in graphene is highly transparent for Dirac electrons

$$\varepsilon_c(\vec{p}) = vp$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{p}} = v\frac{\vec{p}}{p}$$



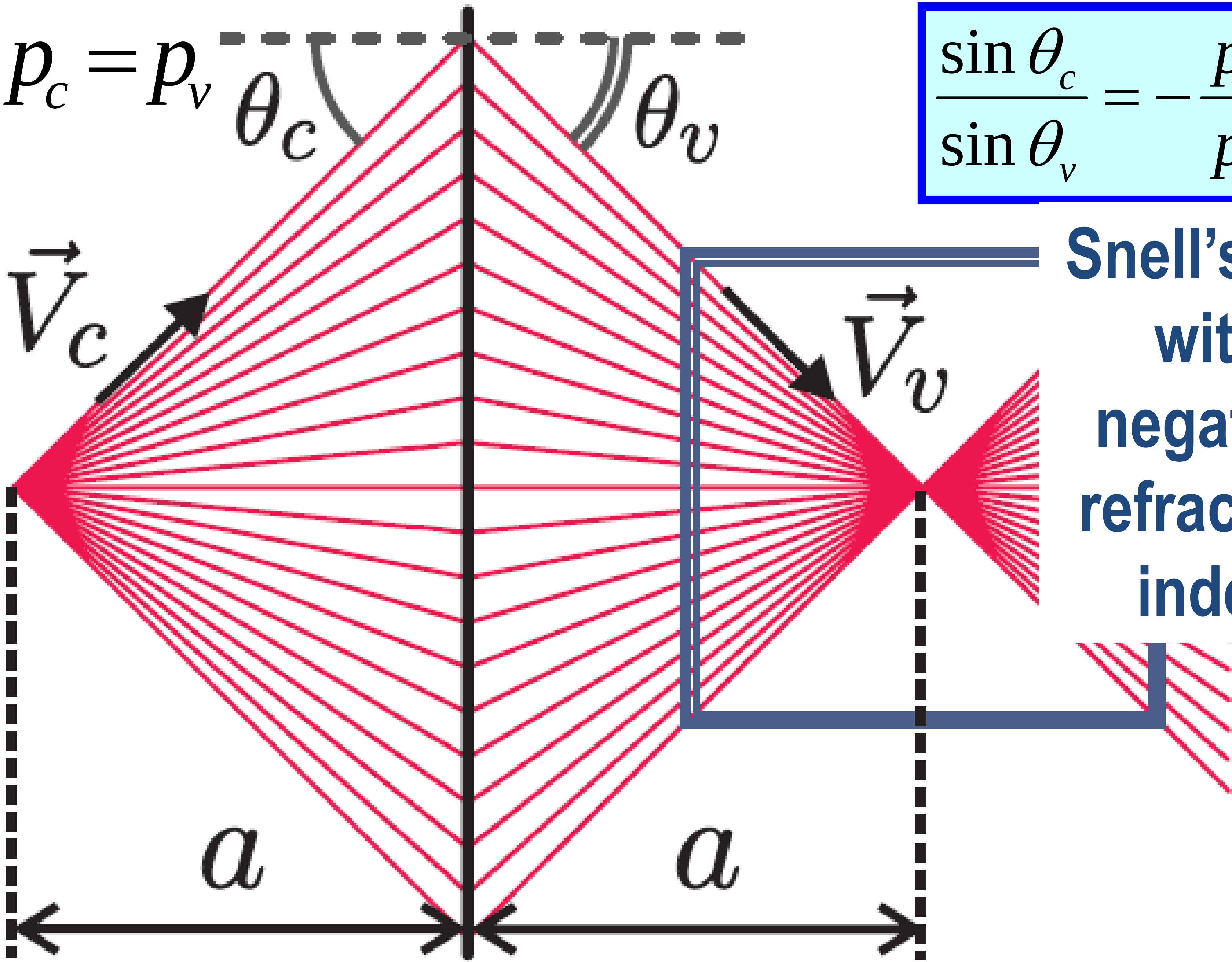
Fermi momentum
p_v

$$\varepsilon_v(\vec{p}) = -vp$$

$$\vec{v} = \frac{\partial \varepsilon}{\partial \vec{p}} = -v\frac{\vec{p}}{p}$$

Cheianov, VF - PR B 74, 041403 (2006)
Katsnelson, Novoselov, Geim, Nature Physics 2, 620 (2006)

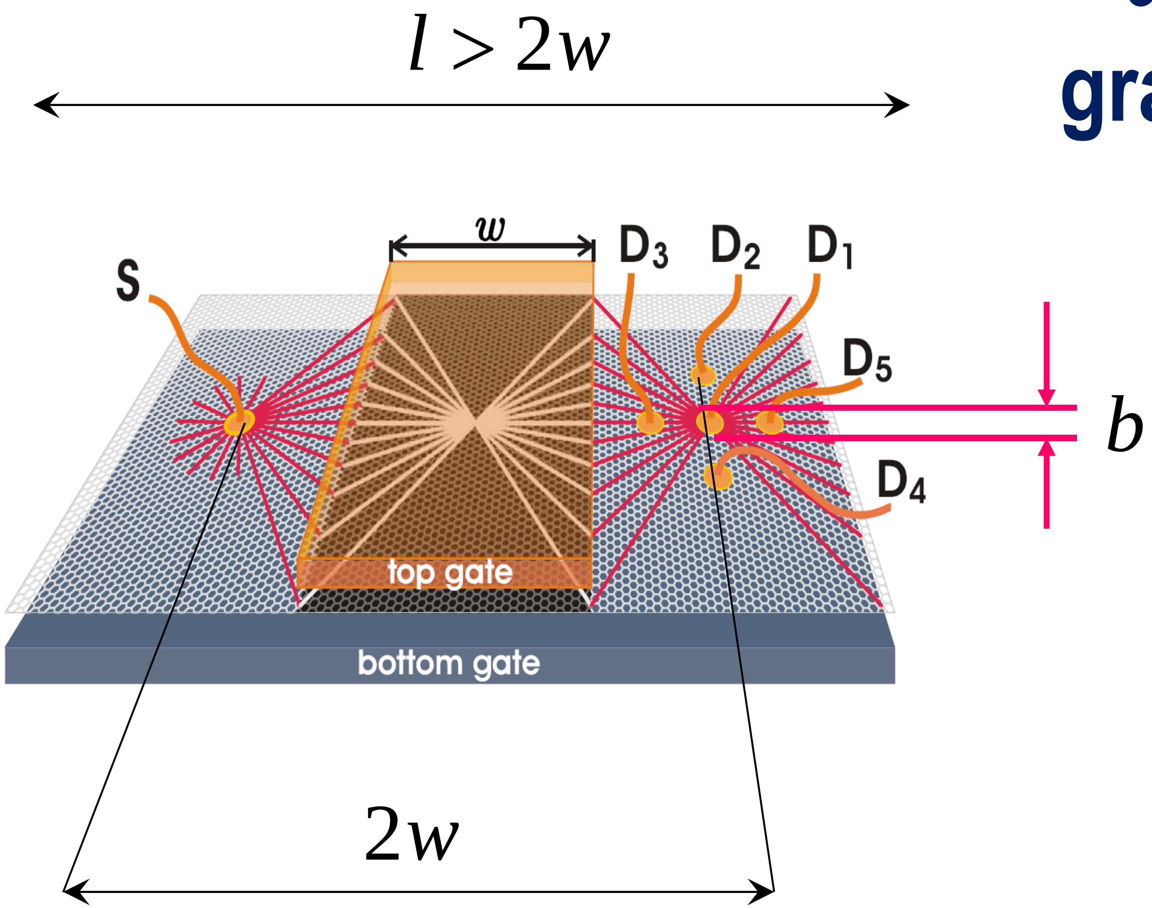
$$\frac{\sin\theta_c}{\sin\theta_v} = -\frac{p_v}{p_c} = n$$



Snell's law with negative refraction index

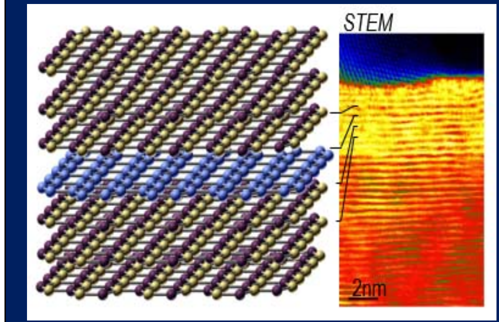
Cheianov, Fal'ko, Altshuler - Science 315, 1252 (2007)

Veselago lens for electrons in ballistic graphene using bipolar PNP graphene transistor



$$kT < \frac{b}{w}\varepsilon_F$$

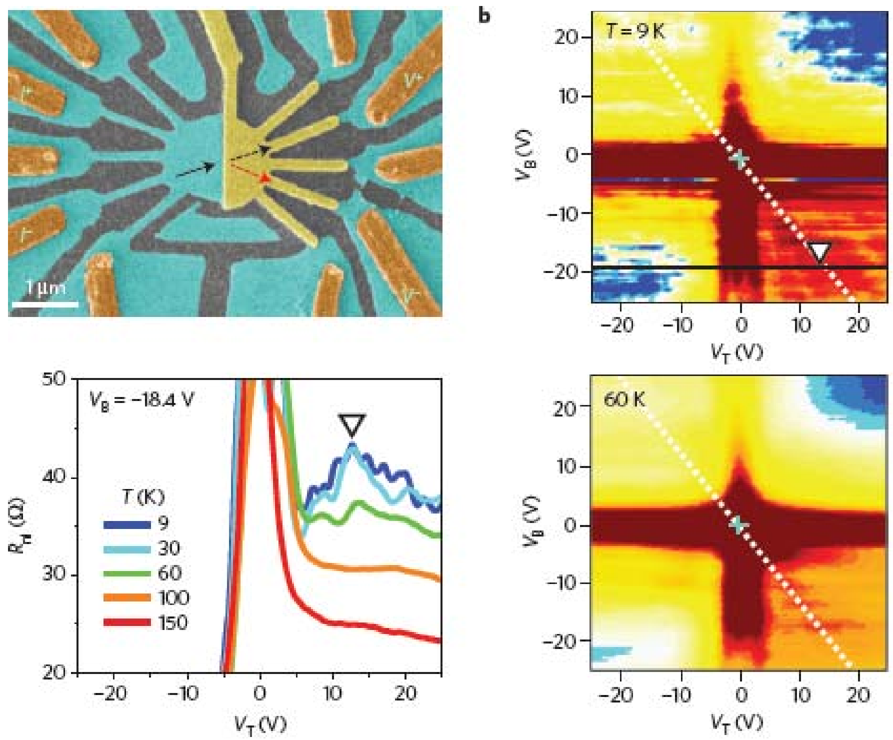
Cheianov, Fal'ko, Altshuler - Science 315, 1252 (2007)

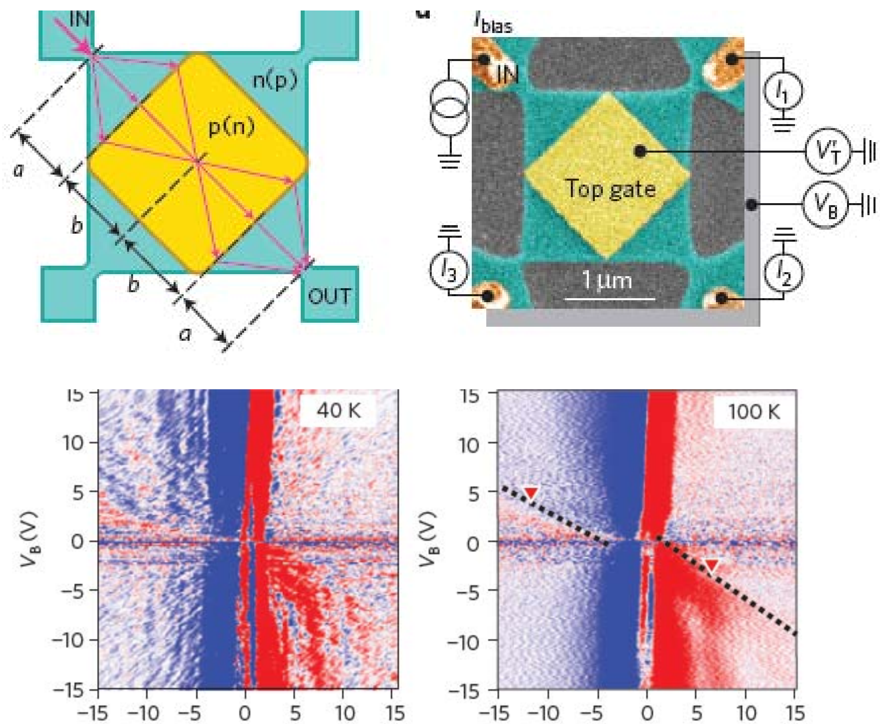


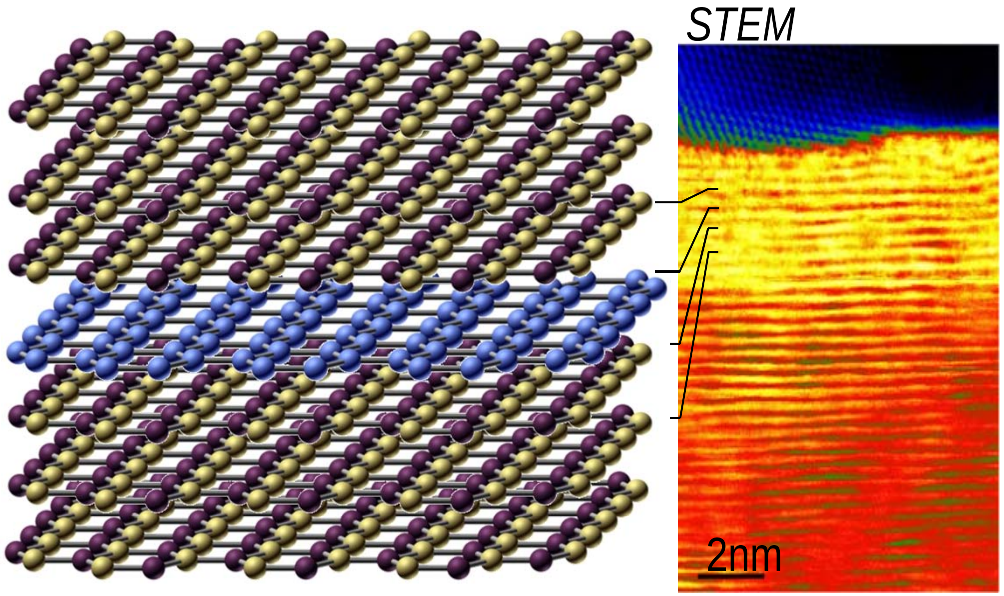
Negative refraction of Dirac electrons in hBN/G/hBN

nature physics
LETTERS
PUBLISHED ONLINE: 14 SEPTEMBER 2015 | DOI: 10.1038/NPHYS3460

Gil-Ho Lee†, Geon-Hyoung Park and Hu-Jong Lee*

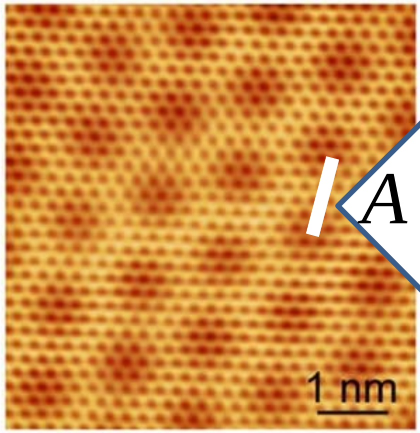






- **Graphene at its best: ballistic electrons in graphene (G) encapsulated in van der Waals heterostructures with hexagonal boron nitride (hBN)**

- **Moiré superlattice in graphene – hBN heterostructures, moiré minibands, and Zak-Brown magnetic minibands**

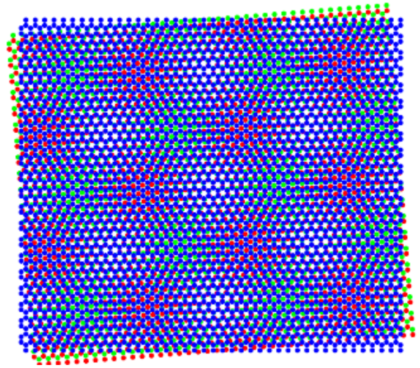


$$A \approx \frac{a}{\sqrt{\delta^2 + \theta^2}}$$

misalignment

lattice mismatch

δ=0.018 for non-strained graphene on hBN

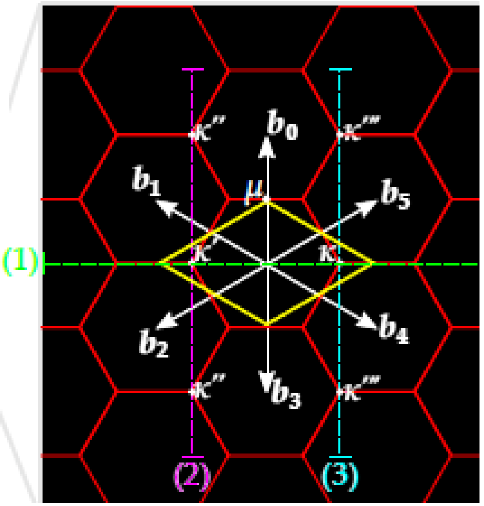


Xue, Sanchez-Yamagishi, Bulmash, Jacquod, Deshpande, Watanabe, Taniguchi, Jarillo-Herrero, LeRoy - Nature Mat 10, 282 (2011)

Long-period moiré patterns are generic for all G/hBN heterostructures, grown and mechanically transferred

Both graphene and hBN lattices are honeycomb,

hence, moiré superlattice is hexagonal

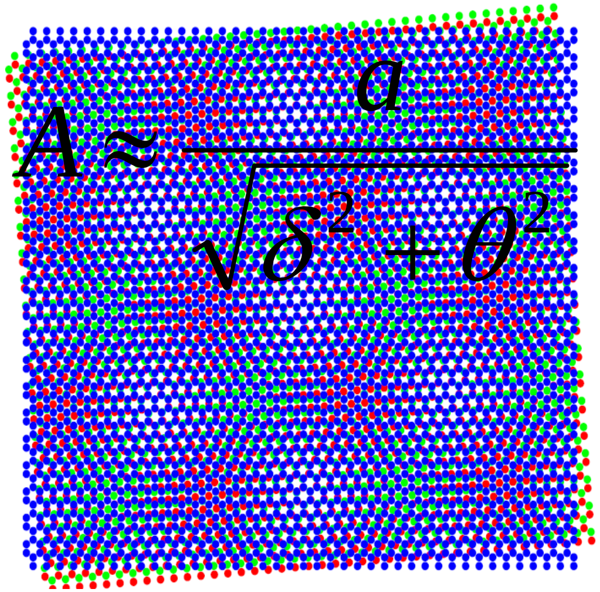


$$\vec{b}_n = \vec{G}_n^{G} - \vec{G}_n^{hBN}$$

$$b_n = \frac{3\pi}{4a}\sqrt{\delta^2 + \theta^2} << K, G^G$$

Due to a separation between layers larger than distance between atoms within the layers, moiré perturbation is dominated by the simplest spatial harmonics

Lopes dos Santos, Peres, Castro Neto - PRL 99, 256802 (2007)
Lopes dos Santos, Peres, Castro Neto - arXiv:1202.1088 (2012)
Bistritzer, MacDonald - PRB 81, 245412 (2010)
Kindermann, Uchoa, Miller - Phys. Rev. B 86, 115415 (2012)

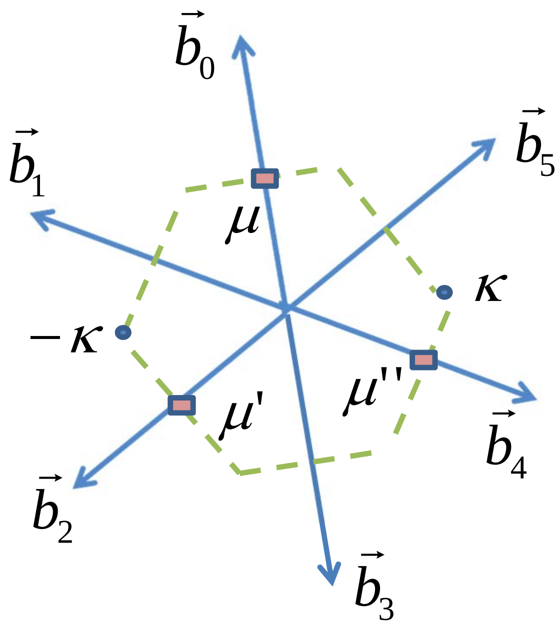


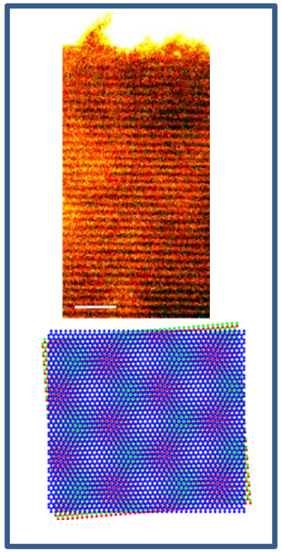
$$\vec{b}_0 = \vec{b}_G - \vec{b}_{BN} = \left[1-(1+\delta)^{-1}\hat{R}_\theta\right]\begin{pmatrix}\frac{4\pi}{3a}\\ 0\end{pmatrix}$$

$$|\vec{b}_0| \equiv b \approx \frac{3\pi}{4a}\sqrt{\delta^2+\theta^2}$$

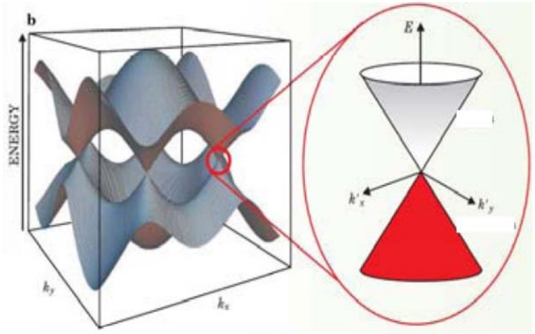
lattice mismatch 1.8% for G/hBN

misalignment $<2^0$





electrons in G/hBN moiré superlattices



$+$

$$a_z > a \Rightarrow \text{only } \vec{b}_m = \vec{G}_m^{G} - \vec{G}_m^{hBN} \longrightarrow \delta H_{moire}$$

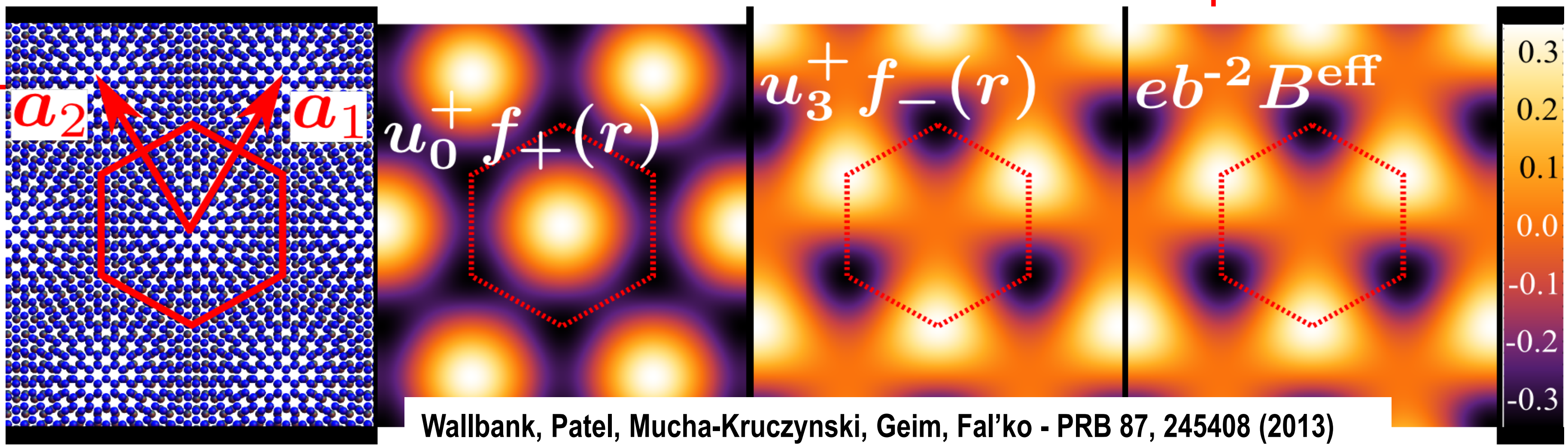
electrostatic modulation

sublattice asymmetry

hopping between sublattices, leading to a pseudomagnetic field

$$\hat{H} = vp\cdot\sigma + u_0 v b f_1(r) + u_3 v b f_2(r)\sigma_3\tau_3 + u_1 v\left[l_z \times \nabla f_2(r)\right]\cdot\sigma\tau_3$$

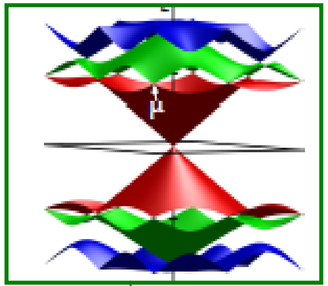
inversion symmetric

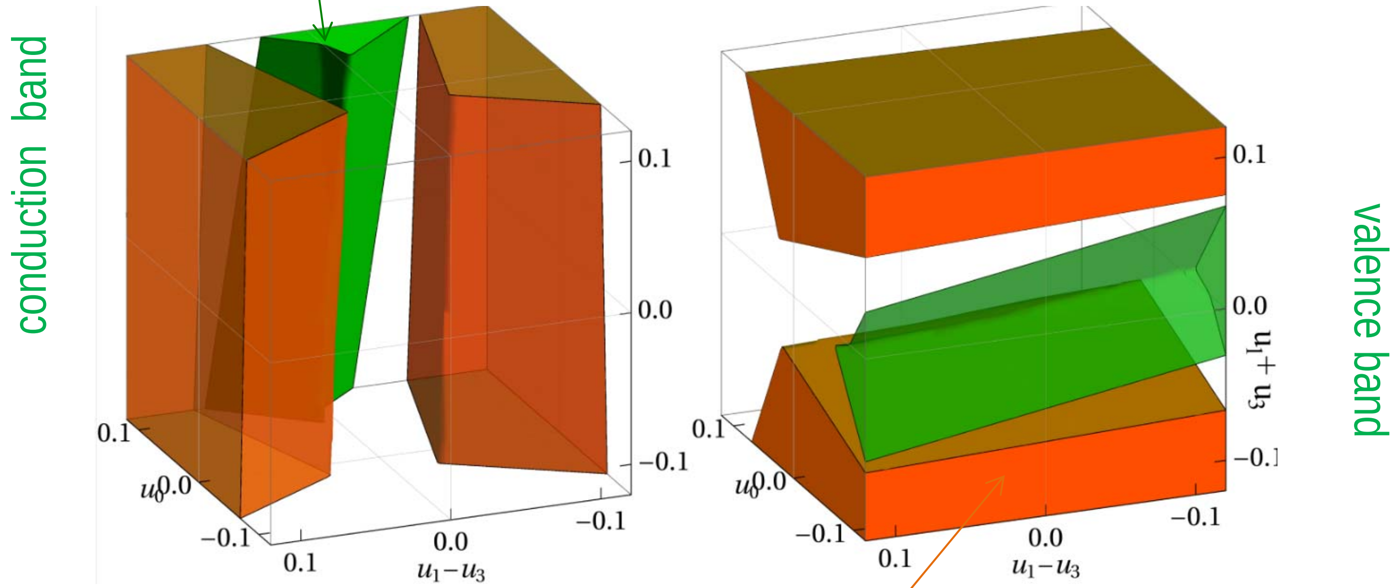


Wallbank, Patel, Mucha-Kruczynski, Geim, Fal'ko - PRB 87, 245408 (2013)

characteristic moiré miniband regimes: no electron-hole symmetry

three mini-DPs at the edge of 1st miniband

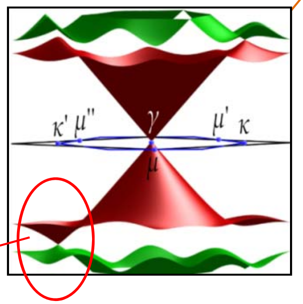




single mini-Dirac point at the edge of 1st miniband

vb

$v_{mDP} \approx \frac{1}{2} v$



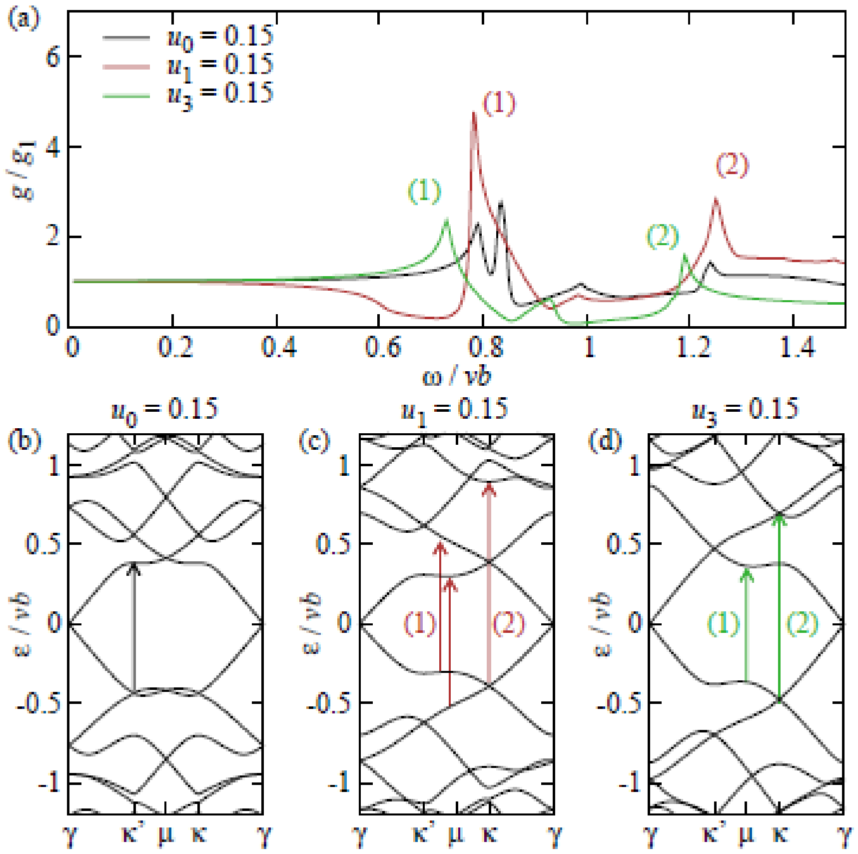
G-hBN hopping model and electric quadrupole moments on, e.g., nitrogen sites

$$vbu_0 \qquad vbu_1 \qquad vbu_3$$

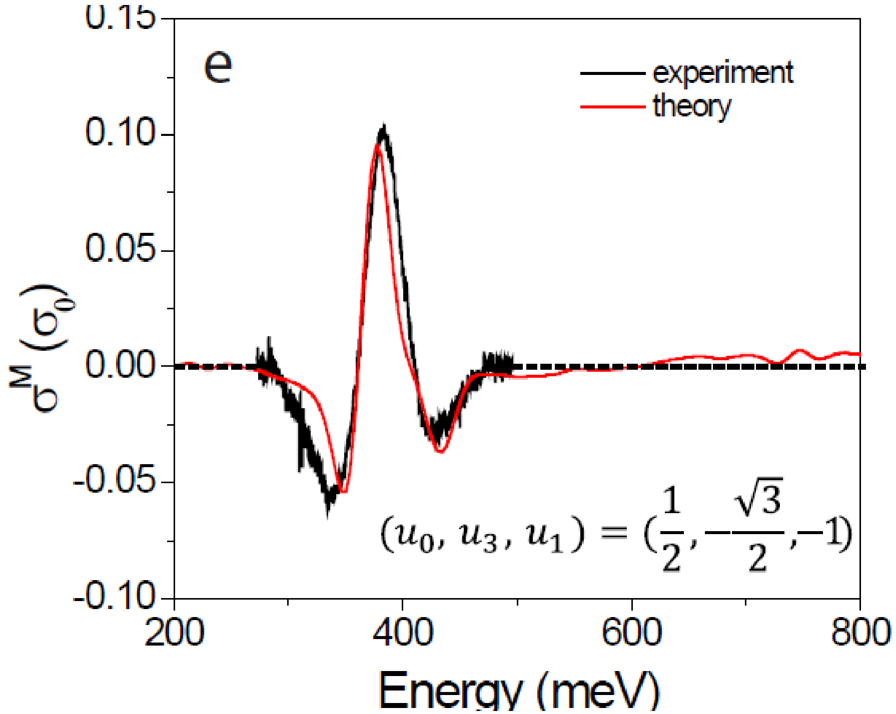
$$\frac{1}{2}u \qquad -\frac{\delta}{\sqrt{\delta^2+\theta^2}}u \qquad -\frac{\sqrt{3}}{2}u$$

Wallbank, Patel, Mucha-Kruczynski, Geim, Fal'ko - PRB 87, 245408 (2013)

Optical signature of moiré minibands

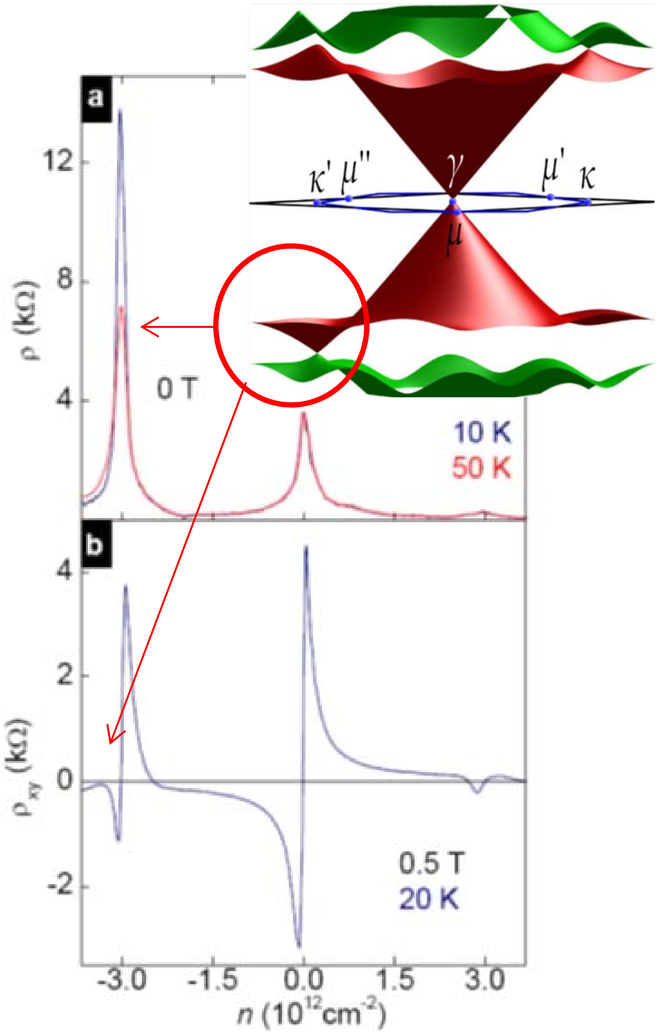


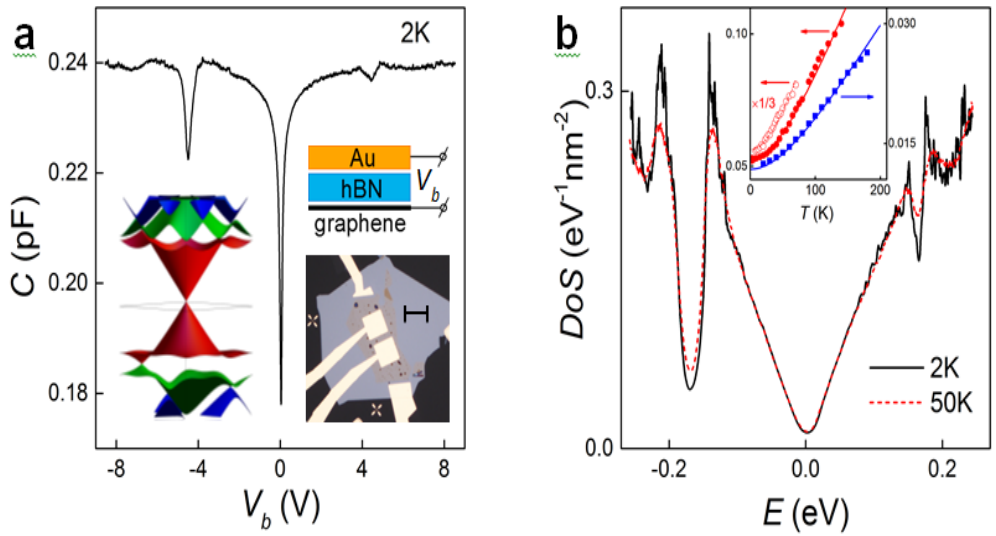
Abergel, Wallbank, Chen, Mucha-Kruczynski, Fal'ko
New J Phys 15, 123009 (2013)



Shi, Jin, Yang, Ju, Horng, Lu, Bechtel, Martin, Fu, Wu, Watanabe, Taniguchi, Zhang, Bai, Wang, Zhang, Wang
arXiv:1405.2032

Manifestation of minibands in magneto-transport and capacitance spectroscopy

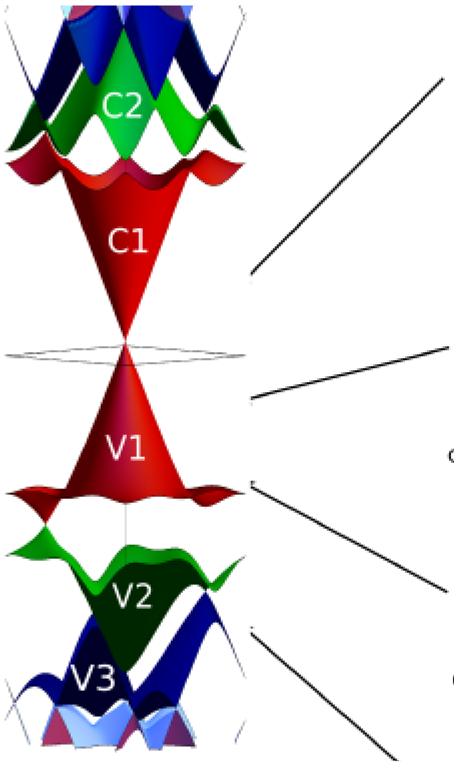


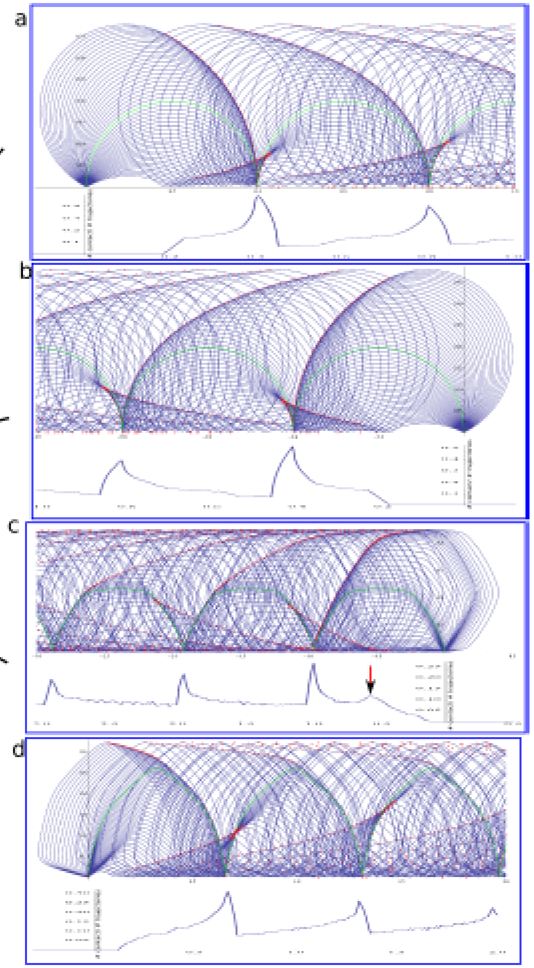


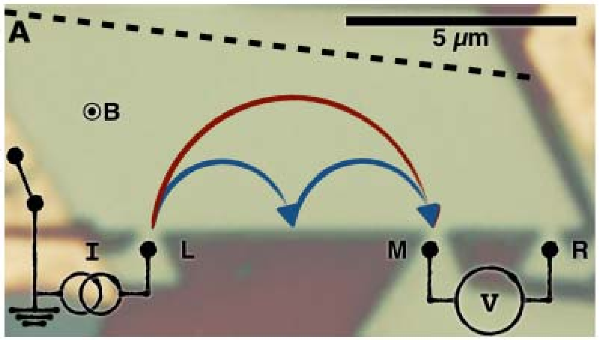
Ponomarenko, Gorbachev, Elias, Yu, Patel, Mayorov, Woods, Wallbank Mucha-Kruczynski, Piot, Potemski, Grigorieva, Guinea, Novoselov, Fal'ko, Geim - Nature 497, 594 (2013)

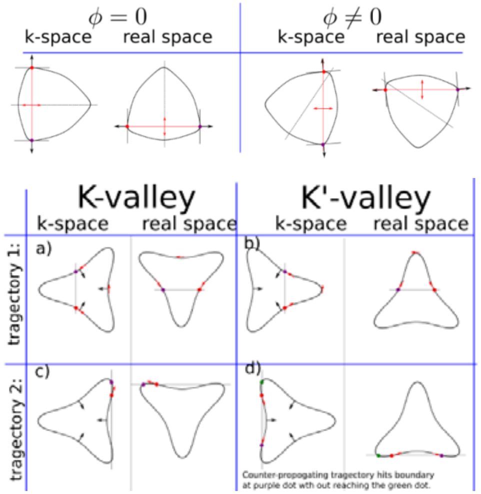
Yu, Gorbachev, Tu, Kretinin, Cao, Jalil, Withers, Ponomarenko, Chen, Piot, Potemski, Elias, Watanabe, Taniguchi, Grigorieva, Novoselov, Fal'ko , Geim, Mishchenko Nature Physics 10, 525 (2014)

Transverse magnetic focusing of electrons in moiré minibands in almost aligned G/hBN









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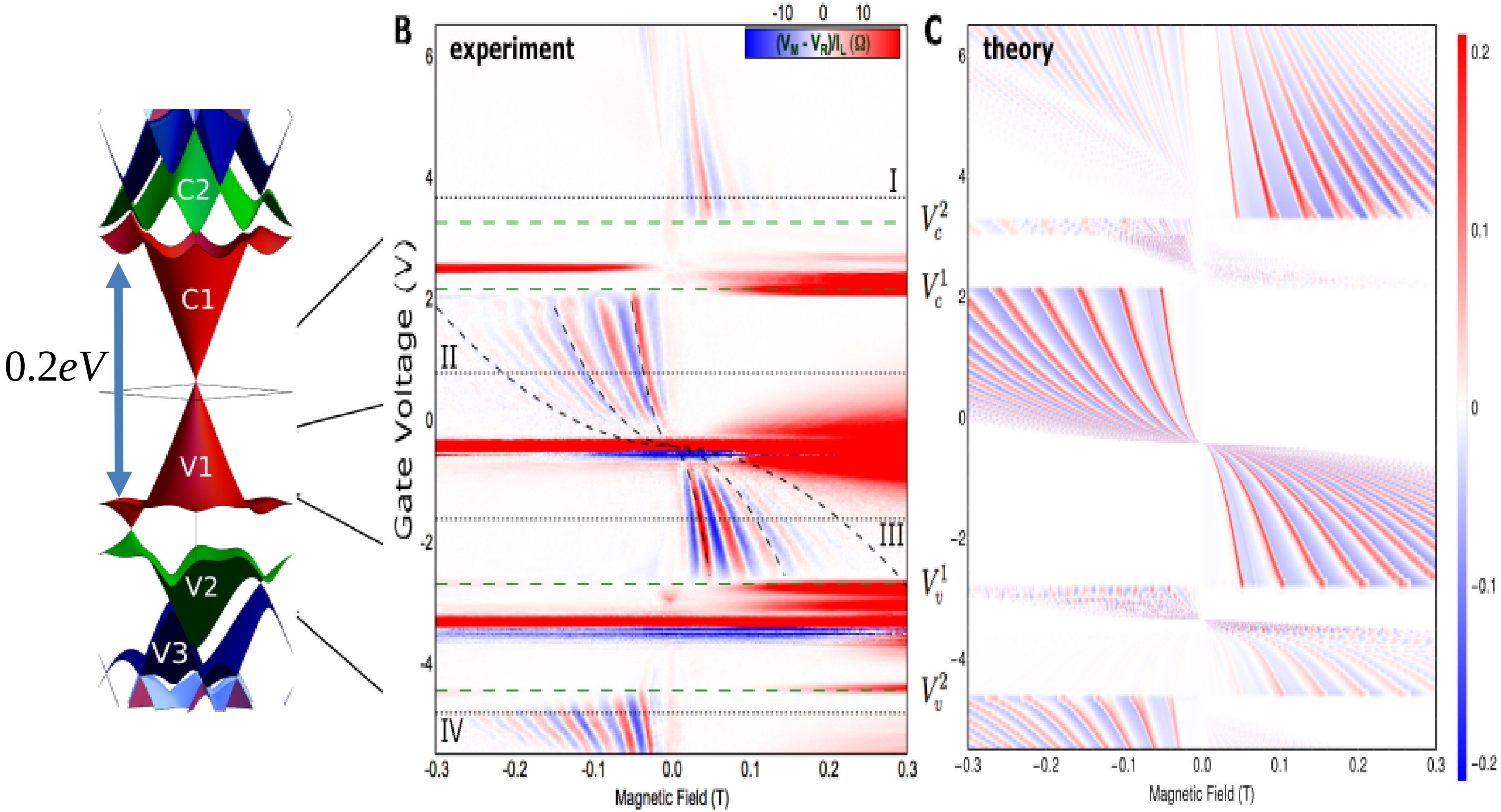


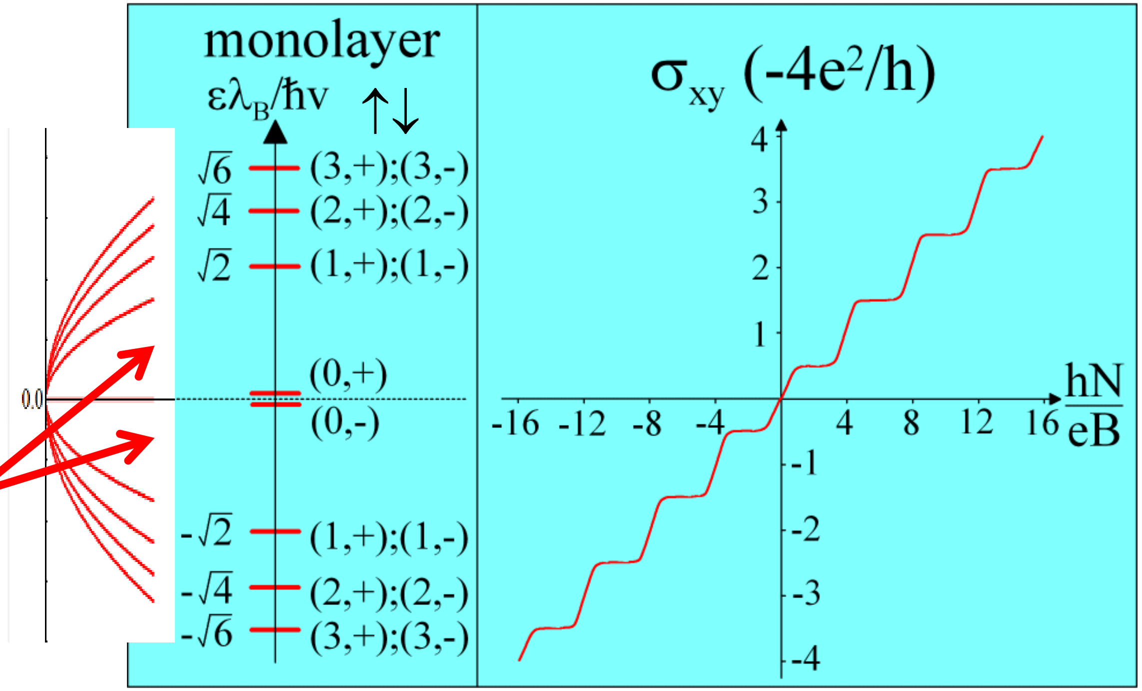
Lee, Wallbank, Gallagher, Watanabe, Taniguchi, Fal'ko, Goldhaber-Gordon - Science 353, 1526 (2016)

Landau levels of Dirac electrons in a magnetic field

$$H = v(\vec{p} - \tfrac{e}{c}\vec{A})\cdot\vec{\sigma} \;\Rightarrow\; \varepsilon^{\pm} = \pm\sqrt{2n}\,\frac{v}{\lambda_B} \propto \sqrt{n\cdot B}$$

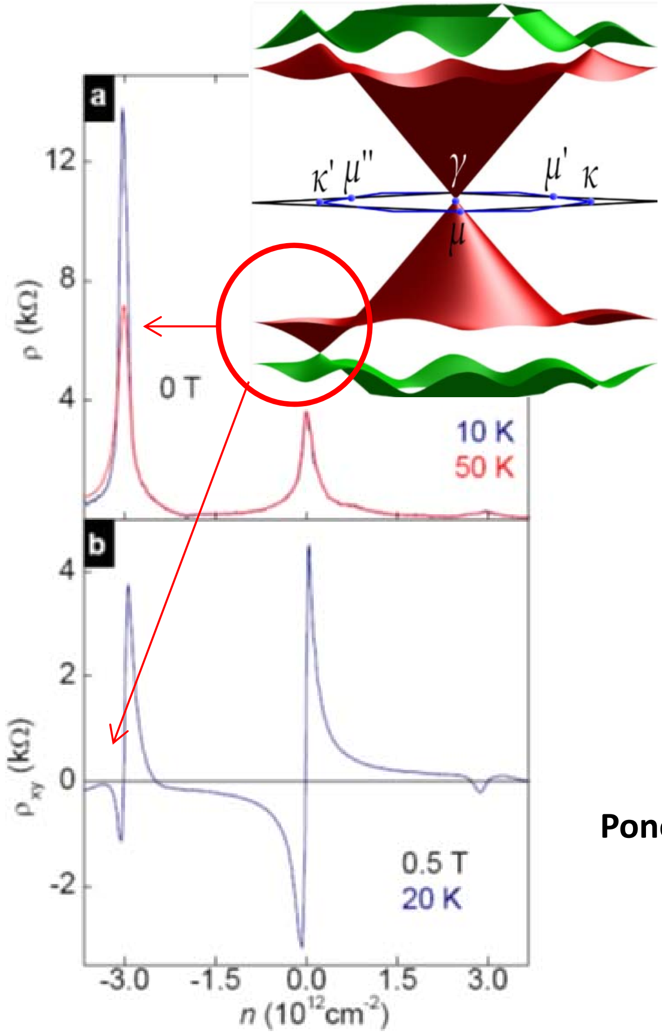
with *4*-fold degenerate Landau level

McClure - Phys. Rev. 104, 666 (1956)

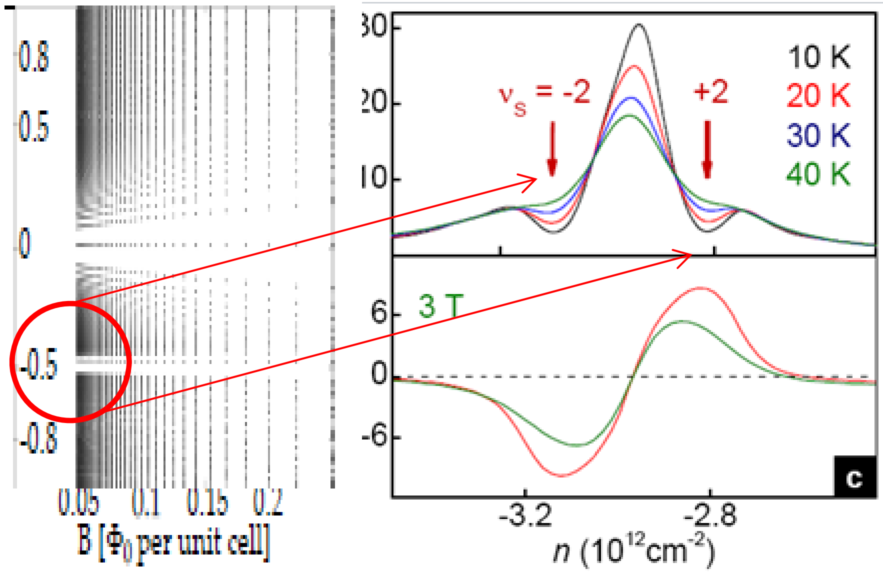


the largest gaps in the LL spectrum

Should be the same for the secondary Dirac electrons at the edge of the 1st moiré miniband



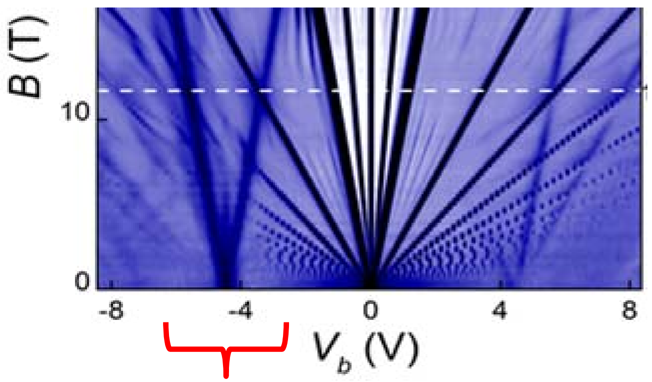
Magneto-transport in oriented graphene-BN heterostructures



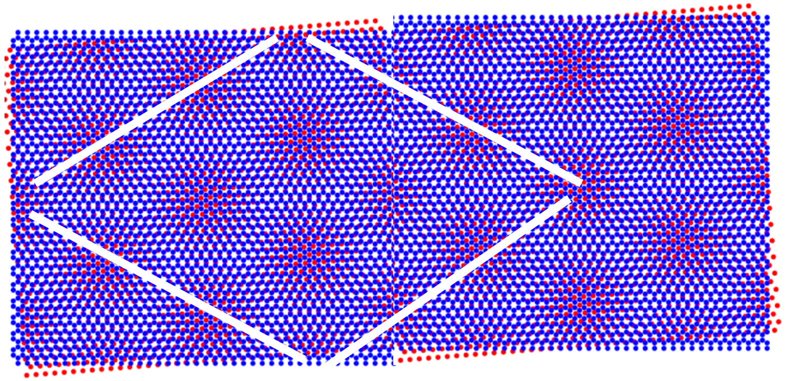
Ponomarenko, Gorbachev, Elias, Yu, Patel, Mayorov, Woods, Wallbank, Mucha-Kruczynski, Piot, Potemski, Grigorieva, Guinea, Novoselov, Fal'ko, Geim Nature 497, 594 (2013)

Magneto-capacitance

Yu, Gorbachev, Tu, Kretinin, Cao, Jalil, Withers, Ponomarenko, Chen, Piot, Potemski, Elias, Watanabe, Taniguchi, Grigorieva, Novoselov, Fal'ko , Geim, Mishchenko Nature Physics 10, 525 (2014)



Brown, PR 133, A1038 (1964); Zak, PR 134, A1602 & A1607 (1964)



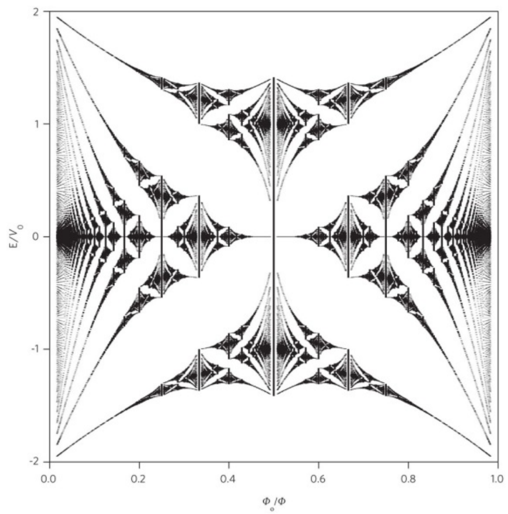
$$\phi \equiv BS = \frac{p}{q}\phi_0, \quad \phi_0 = \frac{h}{e}$$

Magnetic minibands at rational values of magnetic field flux per super-cell

'Magnetic lattice' with a q^2 times bigger effective supercell and q^2 times smaller mini Brillouin zone.

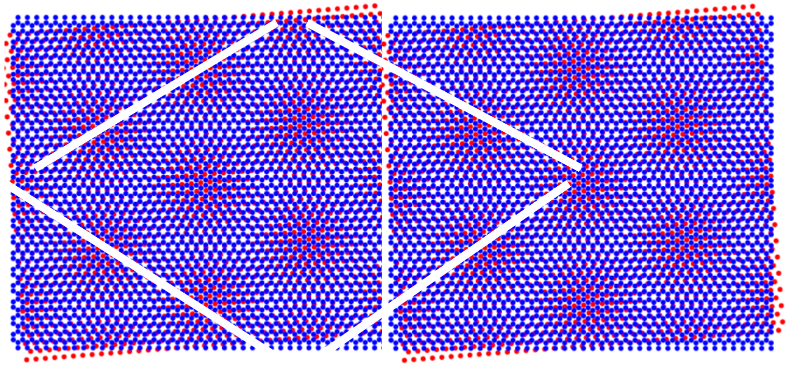
Each state in this mini Brillouin zone is q times degenerate.

Known as fractal 'Hofstadter butterfly' spectrum.



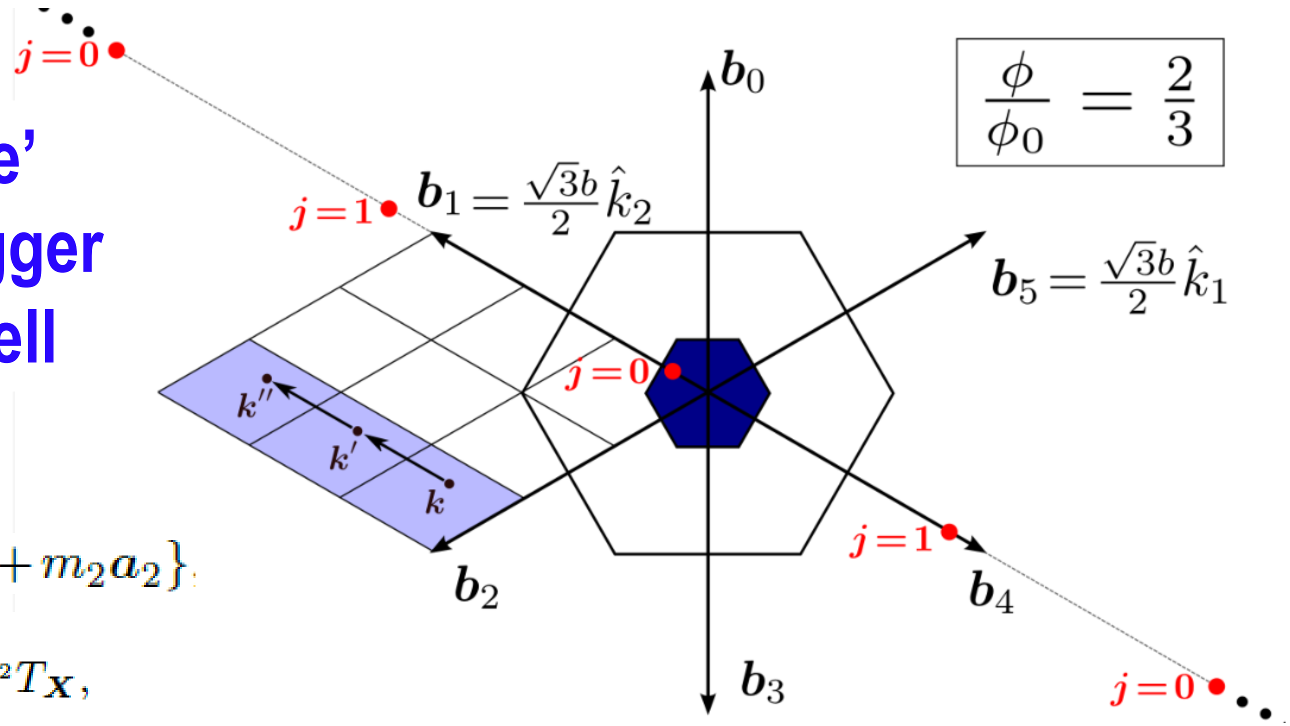
Example for the tight-binding model on a square lattice

Hofstadter PRB 14, 2239 (1976)



Zak-Brown magnetic minibands

'Magnetic lattice' with a *9* times bigger effective supercell



$$G_M = \{\Theta_{\boldsymbol{X}}, \boldsymbol{X} = m_1\boldsymbol{a}_1 + m_2\boldsymbol{a}_2\}$$

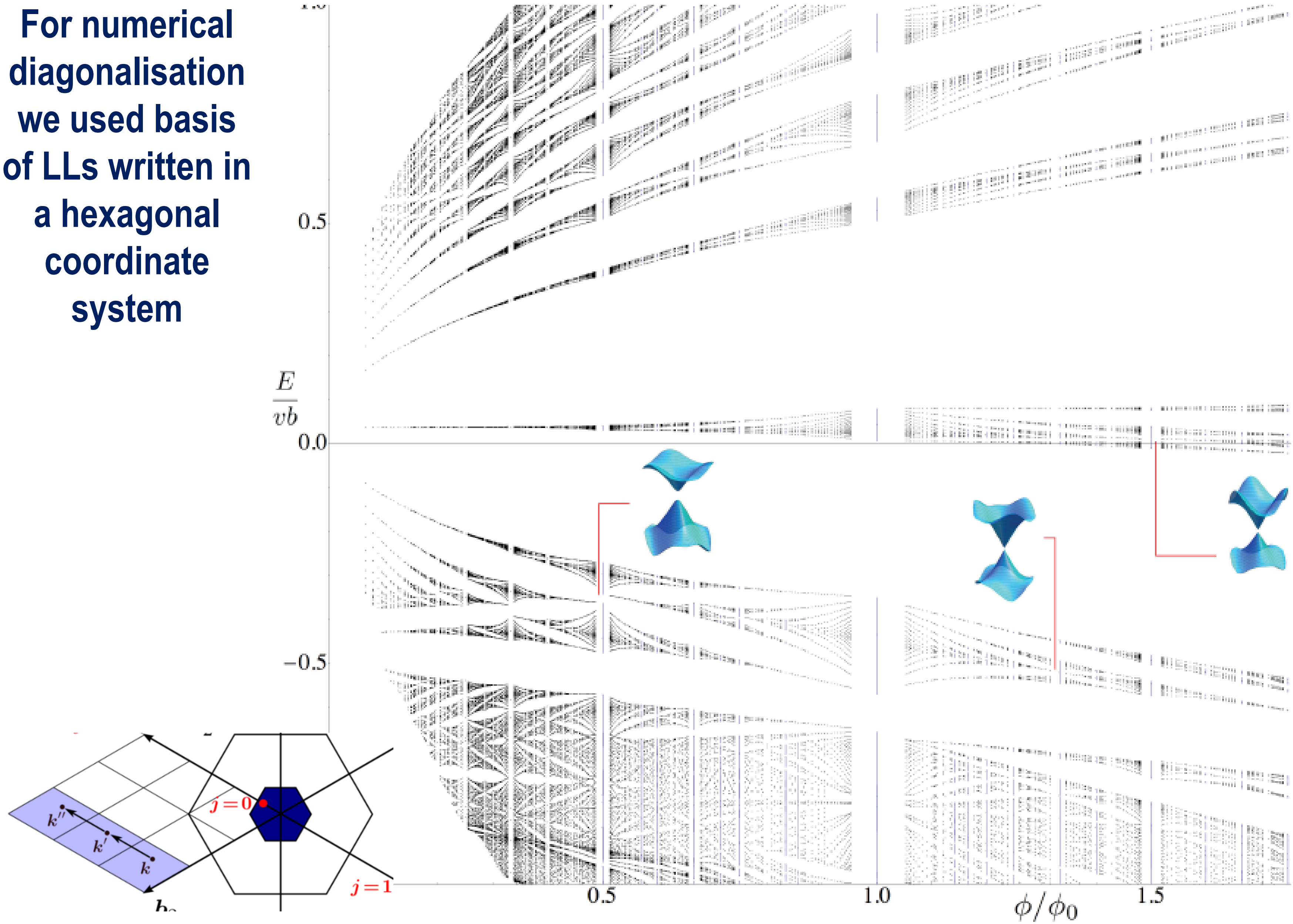
$$\Theta_{\boldsymbol{X}} \equiv e^{-ieBm_1 a\frac{\sqrt{3}}{2}x_2} T_{\boldsymbol{X}},$$

$$\Theta_{\boldsymbol{X}}\Theta_{\boldsymbol{X}'} = e^{-i2\pi\frac{p}{q}m_1' m_2}\Theta_{\boldsymbol{X}+\boldsymbol{X}'},$$

$$\Theta_{\boldsymbol{X}}\Theta_{\boldsymbol{X}'} = e^{-i2\pi\frac{p}{q}(m_1' m_2 - m_1 m_2')}\Theta_{\boldsymbol{X}'}\Theta_{\boldsymbol{X}}.$$

$$G_{qM} = \{\Theta_R, R = qm_1\vec{a}_1 + qm_2\vec{a}_2\} \subset G_M$$

For numerical diagonalisation we used basis of LLs written in a hexagonal coordinate system

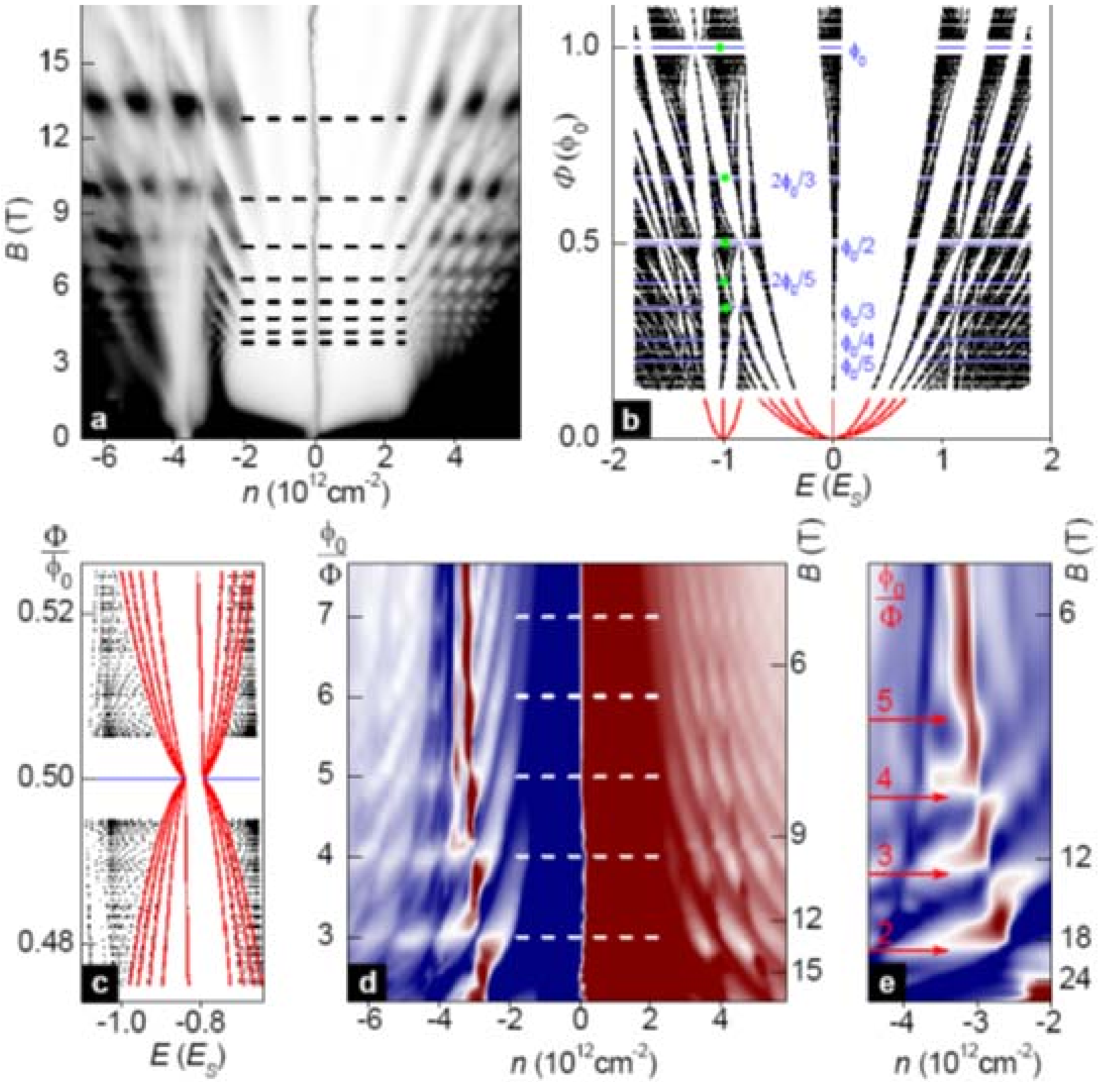


Chen, Wallbank, Patel, Mucha-Kruczynski, McCann, Fal'ko - PRB 89, 075401 (2014)

Low-T magneto-transport in aligned G/BN heterostructures

$$T << \Delta, \hbar\omega_c$$

gaps matter



What is left of such oscillations at high T>>Δ ?

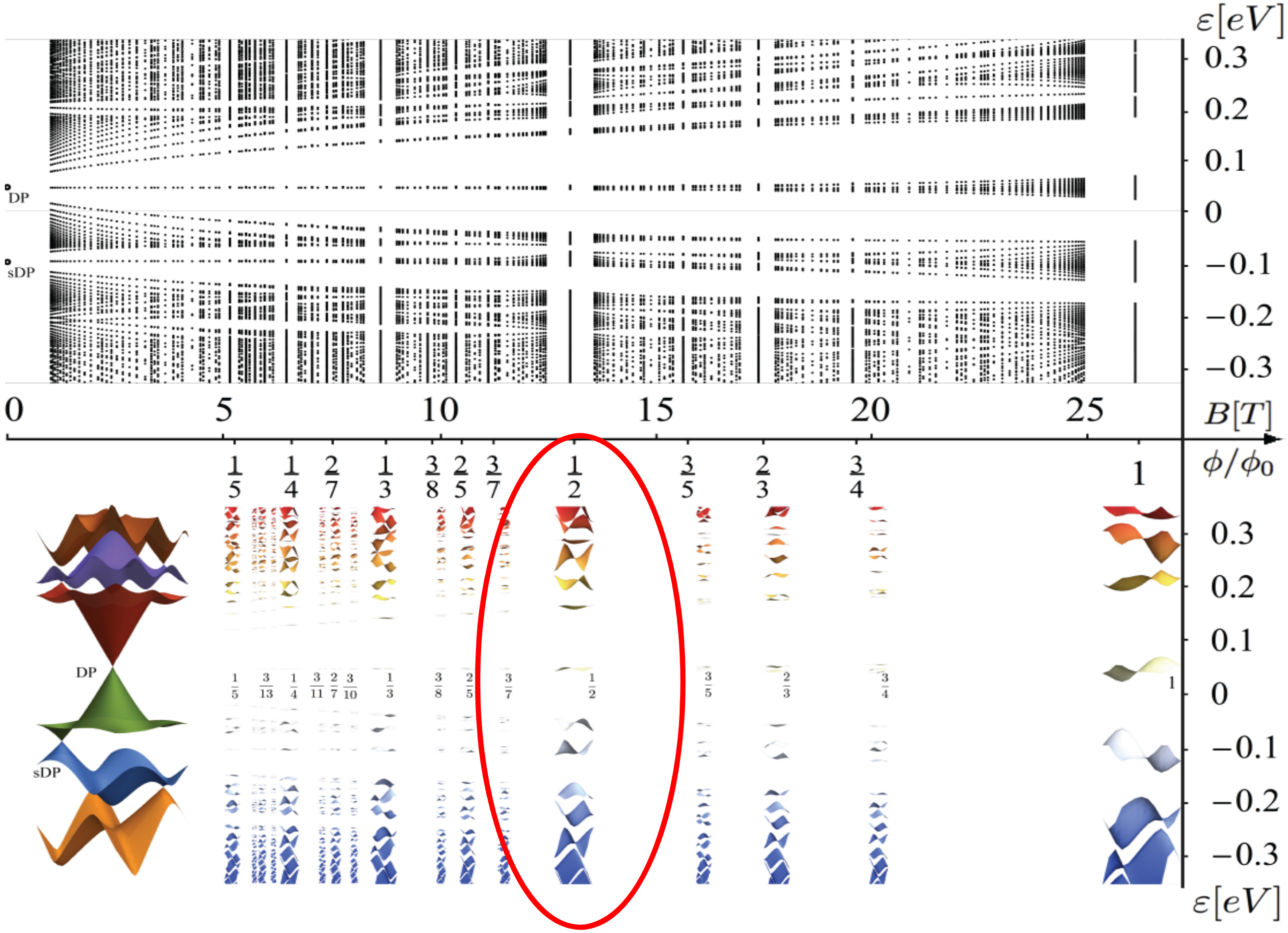
Ponomarenko, Gorbachev, Elias, Yu, Patel, Mayorov, Woods, Wallbank, Mucha-Kruczynski, Piot, Potemski, Grigorieva, Guinea, Novoselov, VF, Geim - Nature 497, 594 (2013)

High-temperature (T>>Δ) Brown-Zak oscillations

Hierarchy of Brown-Zak minibands:

widest minibands at 1/N fractions; then at 2/(2N+1)

all others are much smaller.



$$\sigma_{xx} = \frac{1}{2} e^2 \gamma \tau v^2 \underset{T >> \varepsilon_{band}, \Delta, \hbar\omega}{\Rightarrow} \frac{2e^2}{h} \frac{\varepsilon_F \tau}{\hbar} \frac{\langle v_{bz}^2 \rangle}{v^2}$$

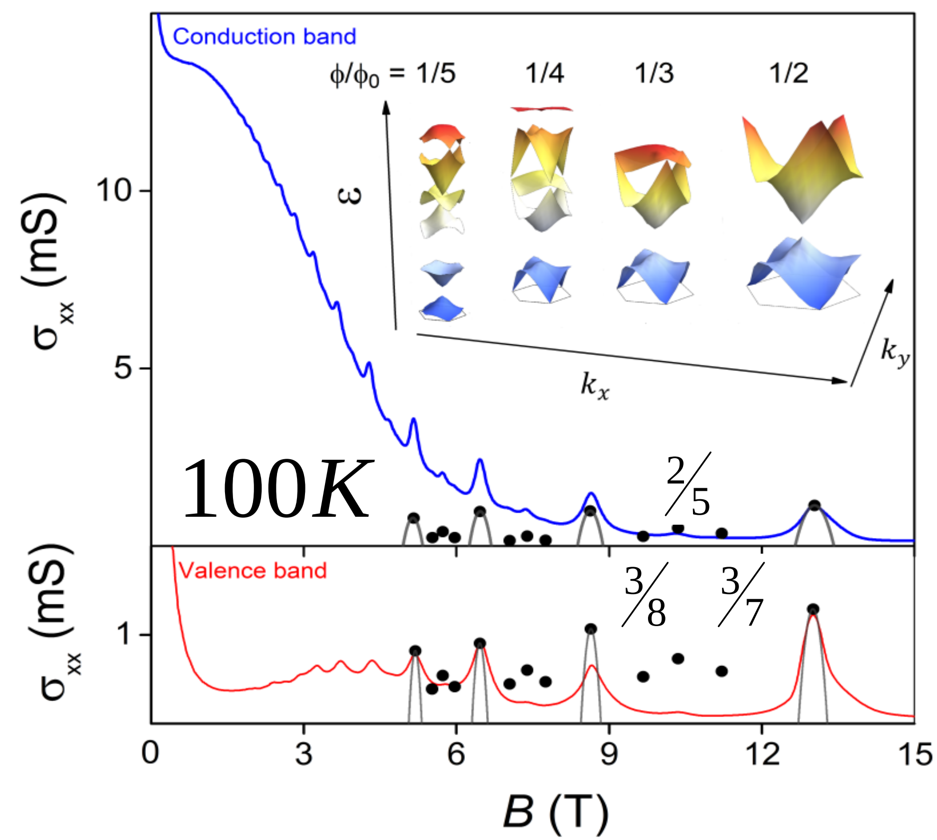
High-temperature Brown-Zak oscillations

$$50 \div 200K >> \varepsilon_{band}, \hbar\omega_c$$

● calculated

$$\sigma_{xx} = \frac{2e^2}{h}\,\frac{\varepsilon_F \tau}{\hbar}\,\frac{\langle v_{bz}^2 \rangle}{v^2}$$

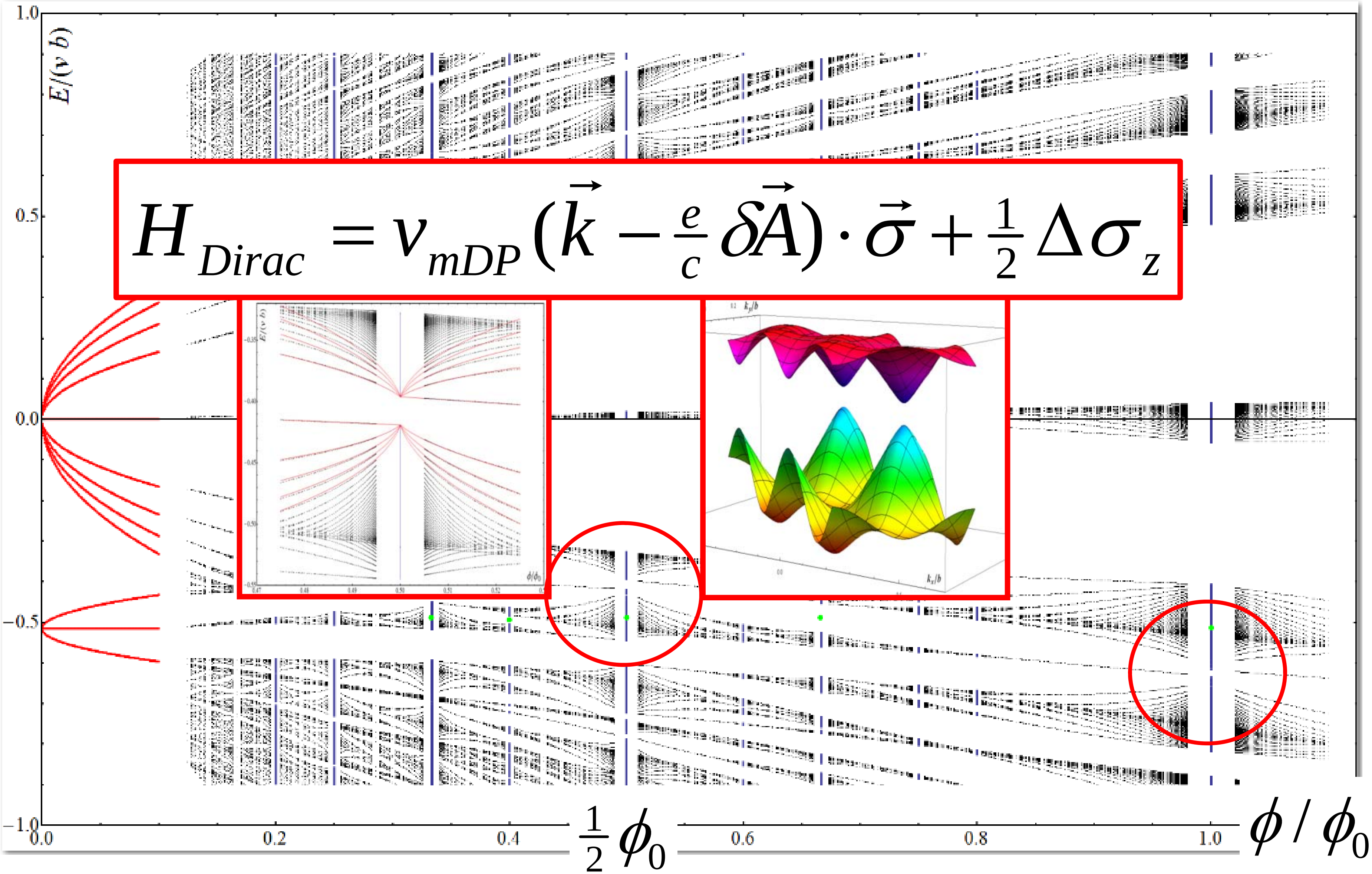
determined by fitting one point, at $\Phi = 1/2\ \Phi_0$



Kumar, Ponomarenko, Geim, Chen, Fal'ko (2016)

Magnetic minibands at $\phi = \frac{p}{q}\phi_0$ - gapped Dirac electrons

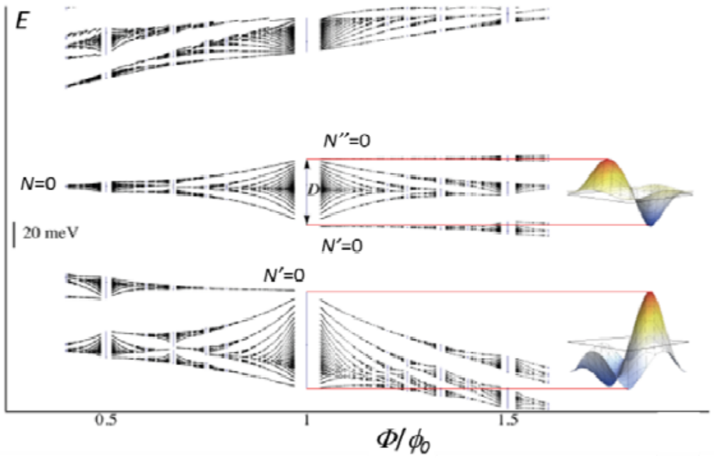
$$H_{Dirac} = v_{mDP}(\vec{k} - \frac{e}{c}\delta\vec{A})\cdot\vec{\sigma} + \frac{1}{2}\Delta\sigma_z$$

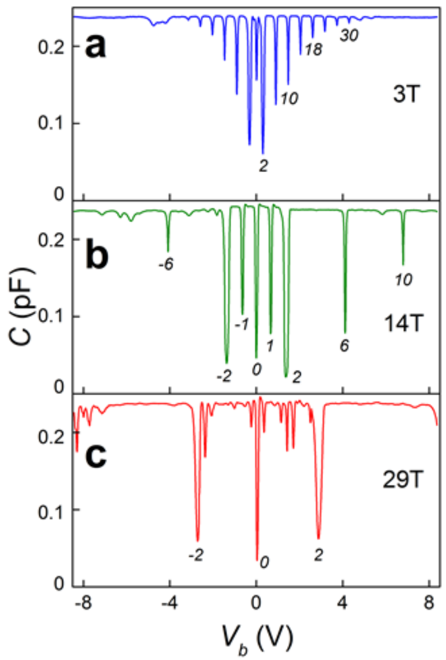


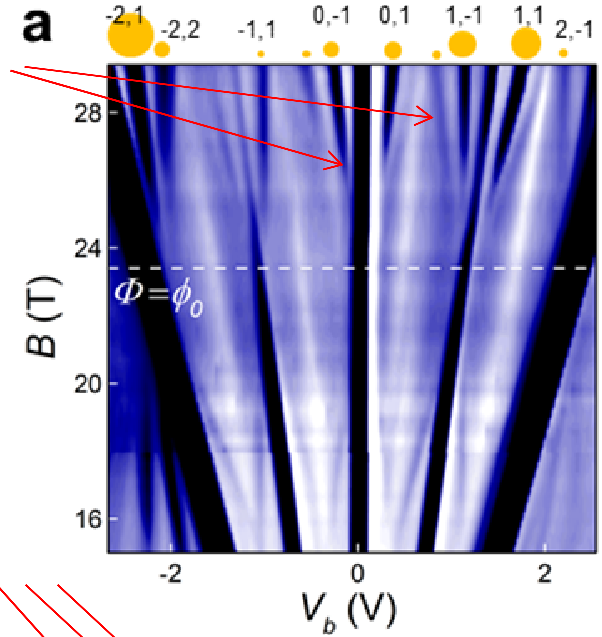
Chen, Wallbank, Patel, Mucha-Kruczynski, McCann, Fal'ko – PRB 89, 075401 (2014)

Capacitance spectroscopy of gaps between magnetic minibands

Yu, Gorbachev, Tu, Kretinin, Cao, Jalil, Withers, Ponomarenko, Chen, Piot, Potemski, Elias, Watanabe, Taniguchi, Grigorieva, Novoselov, VF , Geim, Mishchenko - Nature Physics 10, 525 (2014)



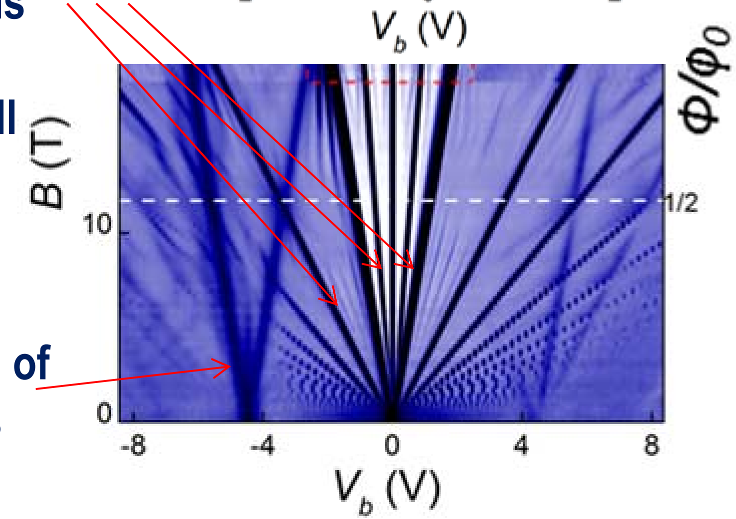




Ferromagnetic quantum Hall effect states in Landau levels of the third generation of Dirac electrons

Gaps between Landau levels and incompressible ferromagnetic quantum Hall states of primary Dirac electrons

Gaps between Landau levels of secondary Dirac electrons




✓ Ballistic electrons in hBN/G/hBN heterostructures
✓ Zak-Brown minibands in g/hBN moiré superlattices: many different 2D metals in one material
Xi Chen (NGI)
John Wallbank (NGI)
Marcin Mucha-Kruczynski (Bath)
Andre Geim (NGI)
Konstantin Novoselov (NGI)
Roman Gorbachev (NGI)
Marek Potemski (CNRS-Grenoble)
David Goldhaber-Gordon (Stanford)
Takashi Taniguchi (NIMS)